# 01 Magnetic Effect of Current and Magnetism

## Moving Charges and Magnetism

The branch of physics which deals with the magnetism due to electric current or moving charge (i.e. electric current is equivalent to the charges or electrons in motion) is called electromagnetism.

### Concept of Field and Oersted Experiment

#### Field

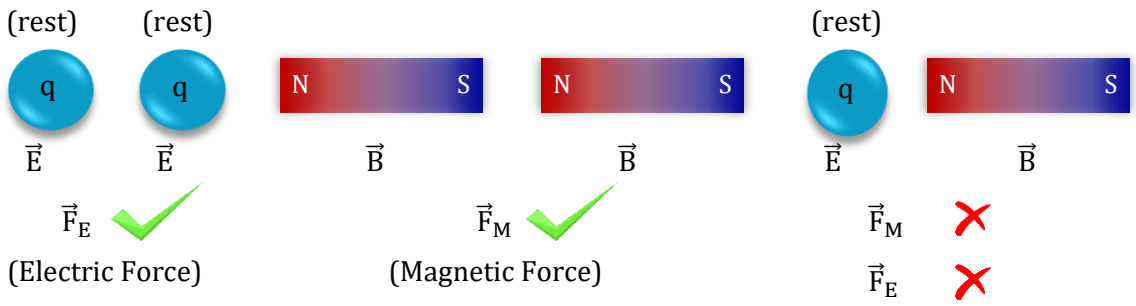
Region around any physical quantity where another similar physical quantity experiences force or torque is called field.

A mass has Gravitational field around it

A charge has Electric field around it

A Magnet has Magnetic field around it

#### The nature of Force




(Electric Force)

(Magnetic Force)

**Illustration 1:**

A charge particle placed near to a current carrying wire then what is nature of force on rest charge?

**Solution:**

A current carrying wire produce magnetic field but charge at rest produces electric field. So no force on charge at rest.

#### Magnetic Field

Region around a magnet or magnetic substance where another magnetic substance experiences force or torque is called Magnetic field.

Unit : SI : weber/(meter)$^2$ or tesla (T)

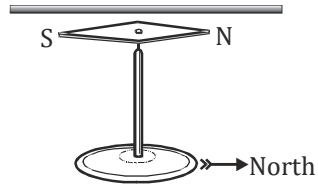
C.G.S. : gauss (1 Tesla = $10^4$ gauss)

Dimensions : $[MT^{-2}A^{-1}]$

## Oersted's Discovery

The relation between electricity and magnetism was discovered by Oersted in 1820. Oersted showed that the electric current through the conducting wire deflects the magnetic needle held near the wire.

- When the direction of current in conductor is reversed then deflection of magnetic needle is also reversed.
- On increasing the current in conductor or bringing the needle closer to the conductor, the deflection of magnetic needle increases.

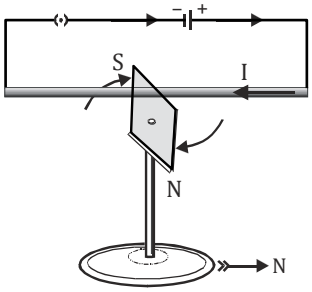



Oersted discovered a magnetic field around a conductor carrying electric current. Other related facts are as follows:

(a) A magnet at rest produces a magnetic field around it while an electric charge at rest produce an electric field around it.

(b) A current carrying conductor has a magnetic field and not an electric field around it. On the other hand, a charge moving with a uniform velocity has an electric as well as a magnetic field around it.

(c) All oscillating or an accelerated charge produces E.M. waves also in additions to electric and magnetic fields.




Oersted's experiment. Current in the wire deflects the compass needle

| Source | Field produced |
|---|---|
| Magnet | Only Magnetic field |
| Charge at rest | Only electric field |
| Current carrying wire | Only Magnetic field |
| Charge in Uniform motion | Both Electric field & Magnetic field |

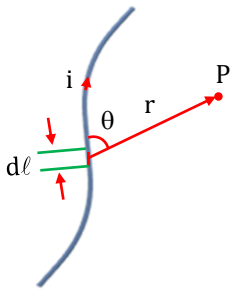
An oscillating/accelerating charge produces:

☑ Electric Field ☑ Magnetic Field ☑ Electromagnetic Waves

## Biot Savarts Law

With the help of experimental results, Biot and Savart arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the field.

**Current element:** A very small element of length dl of a thin current carrying conductor (wire). It is represented by $i(d\vec{\ell})$. It is a vector quantity and its direction is same as that of current.




Experimentally, it was found, magnetic field dB at point P due to current element varies as:

$dB \propto i$

$dB \propto d\ell$

$dB \propto \sin\theta$

$dB \propto \frac{1}{r^2}$

So, $dB \propto \frac{id\ell\sin\theta}{r^2}$

$dB = \frac{\mu_0}{4\pi}\frac{id\ell\sin\theta}{r^2}$ (here $\frac{\mu_0}{4\pi} = 10^{-7}\ \frac{T-m}{A}$)

$\mu_0$ : Permeability of free space (or vacuum)

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### Permeability

Also known as magnetic permeability. It indicates to what extent a material/medium can be magnetized when placed inside an externally applied magnetizing field (to be discussed in Magnetic Materials)

$\mu_0$ : Permeability of free space

$\mu$ : Permeability of a medium

$\mu_r$ : Relative Permeability of a medium

$\mu_r = \frac{\mu}{\mu_0}$ where $\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$

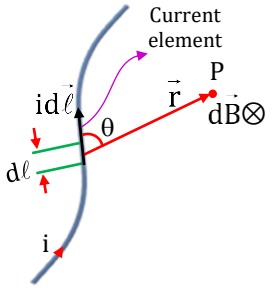
### Vector Form

Experimentally, it is found that direction of magnetic field due to a current element is perpendicular to plane containing $d\vec{\ell}$ and $\vec{r}$

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{id\ell \sin\theta}{r^2} \hat{n}$ ($\hat{n}$ : Unit vector normal to plane containing $d\vec{\ell}$ and $\vec{r}$)

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{(id\ell)(r)\sin\theta}{r^3} \hat{n}$

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\left(d\vec{\ell} \times \vec{r}\right)}{r^3}$




⊗ Inside the plane of paper ◉ Outside the plane of paper

### Illustration 2:

For the magnetic field to the minimum due to a small element of current carrying conductor at a point, the angle between the element and the line joining the element to the given point must be

### Solution:

The magnetic field due to small element of conductor of length is given by $dB = \frac{\mu_0}{4\pi} \frac{Idl\sin\theta}{r^2}$

$$\theta = 0°, \sin\theta = 0$$

### Illustration 3:

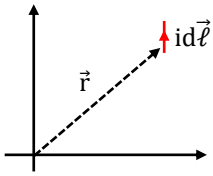
Calculate magnetic field at P

### Solution:

Here $Id\vec{\ell} \| \vec{r}$. So, sinθ = 0. Hence, magnetic field at P is zero.

### Illustration 4:

A small current element $\left(id\vec{\ell}\right)$ is placed at a point P and its position vector is $\vec{r}$. Then find magnetic field at origin.

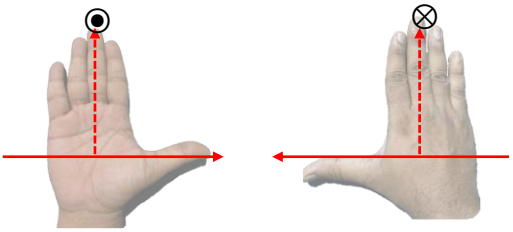



### Solution:

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{\left(\vec{r} \times id\vec{l}\right)}{r^3}$

## Direction of magnetic field

### Right Hand Palm Rule

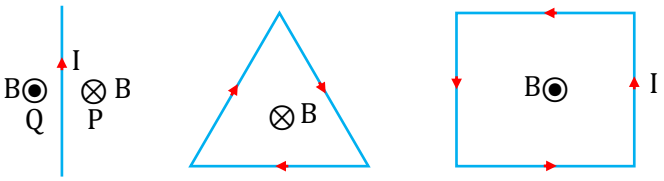


Thumb: along the direction of current

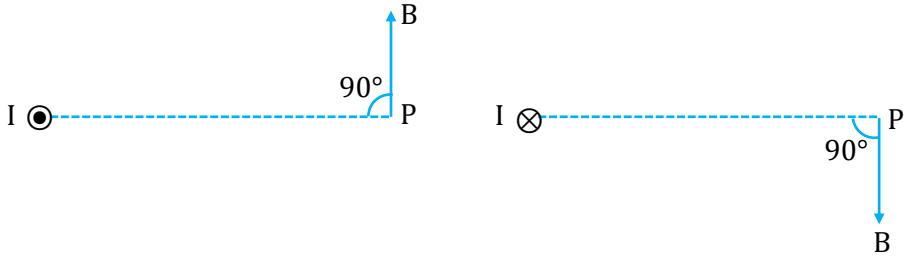
Fingers: along the direction of point where B is to be found

Palm: indicates direction of magnetic field

**Case I :–** Wire is in the plane of the paper :-

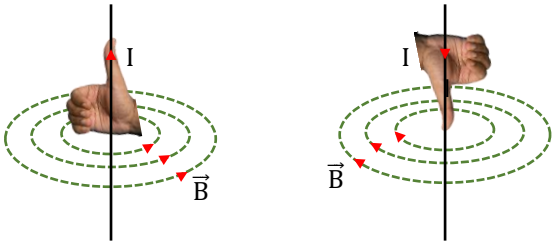



**Case II :–** Wire is perpendicular to the plane of the paper :-




### Right Hand Thumb Rule

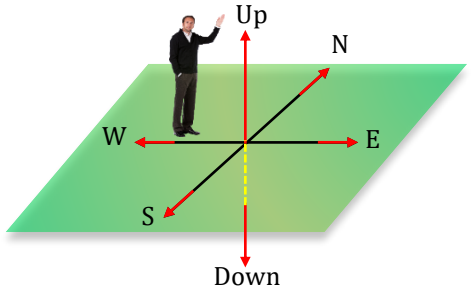
Point the thumb of Right Hand along the direction of current then curling of fingers represents direction of the magnetic field.




**Note:** For a straight current carrying wire M.F. lines are in form of concentric circles.

### Remember!!

East, West, North and South are in Horizontal Plane




### Illustration 5:

A wire has current in South direction. Find direction of magnetic field due to it just below the wire.

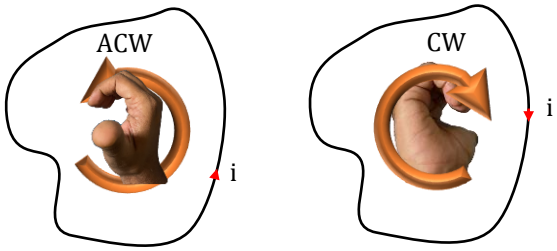
### Solution:

The direction of magnetic field is towards east.

## 1. Due to closed planar loops

- Curl fingers of right hand, in the direction of current in the loop
- Thumb will indicate the direction of magnetic field

(Given loops are in plane of paper)




M.F. → outwards ⊙ M.F. → Inwards ⊗

## Magnetic Field due to Circular Coil

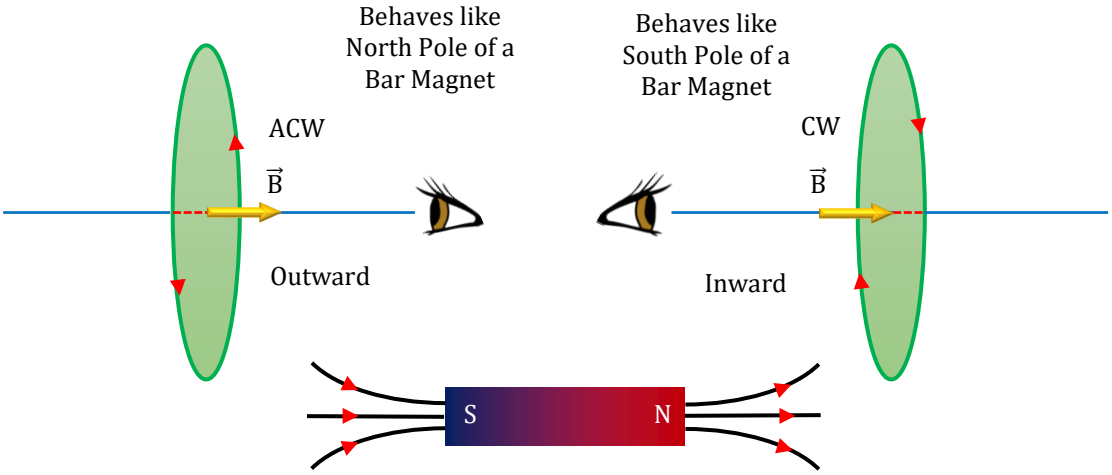
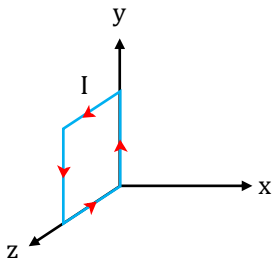



**Illustration 6:**
Which of the following is used to determine the direction of magnetic field in a current carrying conductor?
**Solution:**
The right-hand thumb rule determines the direction of magnetic field in a current carrying conductor. The rule states that when we align our right thumb in the direction of the current and curl our fingers around it, the direction of our fingers is the direction of the magnetic field.
**Illustration 7:**
The relation between the direction of current and the direction of magnetic field is
**Solution:**
When a conductor carries a certain value of current, the current in the conductor and the magnetic field in the conductor are mutually perpendicular to each other.
**Illustration 8:**
Find the direction of magnetic field at point R and S.




**Solution:**
From Right Hand Thumb Rule, direction of magnetic field is along +x axis.

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## Magnetic Field due to moving Charge

MF at a point due to a charge can be found in a way similar to that of a current element.

MF due to current element $d\vec{B}=\frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\vec{r}}{r^3}$

$$Id\vec{\ell}=\left(\frac{dq}{dt}\right)d\vec{\ell}=(dq)\frac{d\vec{\ell}}{dt}=(dq)\vec{v}$$

So, $Id\vec{\ell}=(dq)\vec{v}$

MF due to moving charge $d\vec{B}=\frac{\mu_0}{4\pi}\frac{(dq)\vec{v}\times\vec{r}}{r^3}$

For moving charge $\vec{B}=\frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3}$ (in vector form)

$B=\frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2}$ (in scalar form)

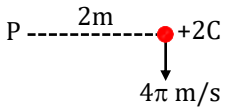
### Direction:

For Positive charge flow :- same as current element (Right Hand Rule)

For Negative charge flow :- Opposite to current element.

### Illustration 9:

A charge +2C moves vertically downward with speed 4πm/s. Find magnetic field at point P.

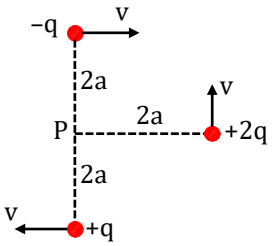



### Solution:

$$B=\frac{\mu_0 qv}{4\pi r^2}\Rightarrow B=\frac{\mu_0}{4\pi}\frac{(2)(4\pi)}{4}\otimes\Rightarrow B=\frac{\mu_0}{2}\otimes\text{ Tesla}$$

### Illustration 10:

A charge +2q moves vertically upwards with sped v, a second charge –q moves horizontally to the right with the same speed v, and a third charge +q moves horizontally to the left with the same speed v. The point P is located at a perpendicular distance a away from each charge as shown in the figure. The magnetic field at point p is




### Solution:

We know that magnetic field due to a moving charge

$$\vec{B}=\frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3}\quad B=\frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2}$$

$$\vec{B}_P=\vec{B}_q+\vec{B}_{2q}+\vec{B}_{-q}$$

$$=\frac{\mu_0}{4\pi}\frac{qv}{(2a)^2}\otimes+\frac{\mu_0}{4\pi}\frac{2qv}{(2a)^2}\odot+\frac{\mu_0}{4\pi}\frac{qv}{(2a)^2}\odot=\frac{\mu_0}{8\pi}\frac{qv}{a^2}\odot$$

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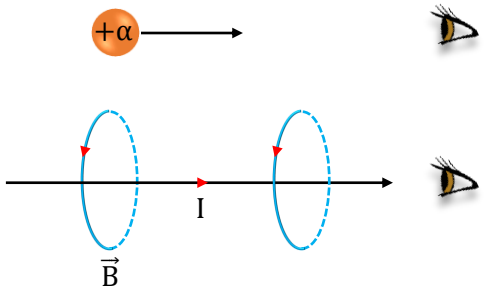
### Illustration 11:

A $\alpha$ – particle moves along horizontal line towards observer. What will be the pattern of concentric circular field lines of magnetic field which produced due to its motion.

### Solution:

Concentric circular field lines will be anti-clockwise as seen by observer.



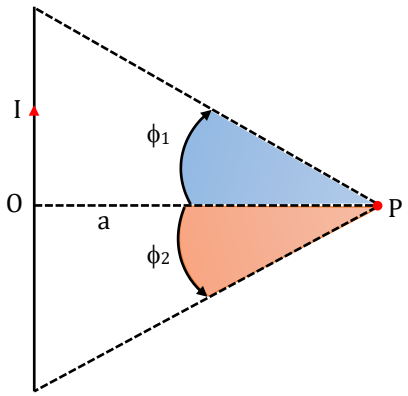

## Application of Biot-Savart Law

### Magnetic Field due to Thin Wire of Finite Length

**AIM:** To find magnetic field due to a current carrying wire of finite length at a point P.

**Geometry:** Point P is at a perpendicular/shortest distance 'a' from the wire.

Ends of the wire make an angle $\phi_1$ and $\phi_2$ with line OP




**Result:** $B = \frac{\mu_0 I}{4\pi a}(\sin\phi_1 + \sin\phi_2)$

$\phi_2 \rightarrow$ Start Point of current

$\phi_1 \rightarrow$ End Point of current

Let's take a current element $Id\ell$ at a distance $\ell$ from point O of the wire.

$$\tan\theta = \frac{\ell}{a} \Rightarrow \ell = a\tan\theta$$

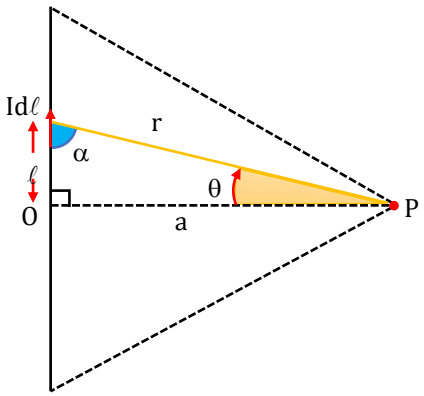
differentiating on both sides,

$$\Rightarrow d\ell = a\sec^2\theta d\theta$$

If distance of current element from point P is r,

$$\cos\theta = \frac{a}{r} \Rightarrow r = \frac{a}{\cos\theta} \Rightarrow r = a\sec\theta$$

and, $\alpha = 90° - \theta$




M.F. at P due to current element: -

$$dB = \frac{\mu_0}{4\pi}\frac{(Id\ell)\sin(180° - \alpha)}{r^2} = \frac{\mu_0}{4\pi}\frac{(Id\ell)\sin\alpha}{r^2}$$

$$\sin\alpha = \sin(90° - \theta) = \cos\theta$$

$$d\ell = a\sec^2\theta d\theta$$

$$r = a\sec\theta$$

$$\Rightarrow dB = \frac{\mu_0}{4\pi}\frac{I\left(a\sec^2\theta d\theta\right)\cos\theta}{a^2\sec^2\theta}$$

$$\Rightarrow dB = \frac{\mu_0}{4\pi}\frac{I\cos\theta d\theta}{a}$$

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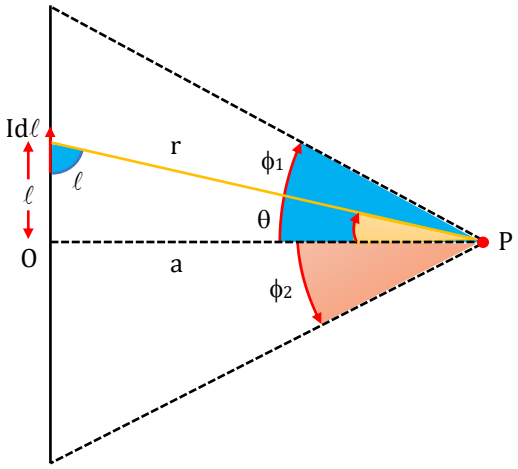
M.F. at P due to current element: -

$$\Rightarrow dB = \frac{\mu_0}{4\pi}\frac{I\cos\theta d\theta}{a}$$

Due to all such current elements, M.F. will be inward to plane of paper,

So, net M.F. will be sum of all such elements




$$\Rightarrow B_{net} = \int \frac{\mu_0}{4\pi}\frac{I\cos\theta d\theta}{a}$$

here, $\theta$ varies from $-\phi_2$ to $\phi_1$

$$\Rightarrow B_{net} = \frac{\mu_0 I}{4\pi a}\int_{-\phi_2}^{\phi_1}\cos\theta d\theta$$

M.F. at P due to straight current carrying wire

$$\Rightarrow B_{net} = \frac{\mu_0 I}{4\pi a}\int_{-\phi_2}^{\phi_1}\cos\theta d\theta$$

$$\Rightarrow B_{net} = \frac{\mu_0 I}{4\pi a}(\sin\theta)\Big|_{-\phi_2}^{\phi_1}$$

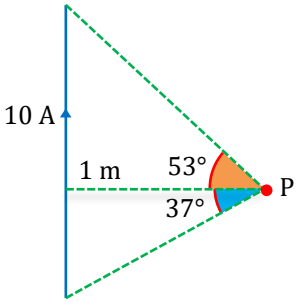
$$\Rightarrow B_{net} = \frac{\mu_0 I}{4\pi a}(\sin\phi_1 - \sin(-\phi_2))$$

$$\Rightarrow B_{net} = \frac{\mu_0 I}{4\pi a}(\sin\phi_1 + \sin\phi_2)$$

Direction : By right hand palm rule, inwards

### Illustration 12:

Find magnetic field at point P.




### Solution:

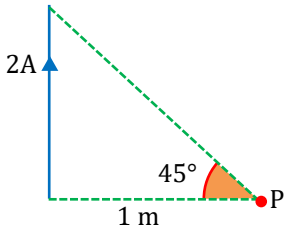
$$B = \frac{\mu_0 I}{4\pi a}(\sin\phi_1 + \sin\phi_2)$$

$$= \frac{10^{-7}(10)}{(1)}\left[\frac{4}{5} + \frac{3}{5}\right]$$

$$= 10^{-6}\left(\frac{7}{5}\right) = \frac{7}{5}\times 10^{-6}\,T\otimes$$

### Illustration 13:

Find magnetic field at point P.




### Solution:

$$B = \frac{\mu_0 I}{4\pi a}(\sin\phi_1 + \sin\phi_2)$$

$$= \frac{10^{-7}(2)}{(1)}\left[\frac{1}{\sqrt{2}} + 0\right]$$

$$= \sqrt{2}\times 10^{-7}\,T\otimes$$

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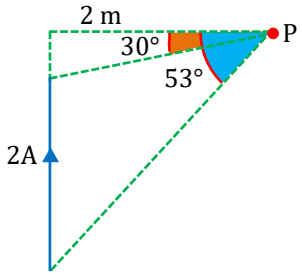
### Illustration 14:

Find magnetic field at point P.




### Solution:

$\phi_2 = 53°, \phi_1 = -30°$

$$B = \frac{\mu_0 I}{4\pi a}(\sin\phi_1 + \sin\phi_2)$$

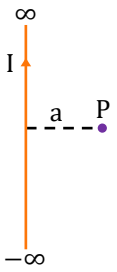
$$= \frac{10^{-7}(20)}{(2)}\left[-\frac{1}{2} + \frac{4}{5}\right]$$

$= 10^{-6}(0.3) = 3 \times 10^{-7}\ T\otimes$

## Magnetic Field due to Infinite Straight Wire

$$B = \frac{\mu_0 I}{4\pi a}(\sin\phi_1 + \sin\phi_2)$$

$\phi_2 \rightarrow$ Start Point of current, $\phi_1 \rightarrow$ End Point of current




Here,

$\phi_1 \rightarrow 90°,\ \phi_2 \rightarrow 90°$

$$B = \frac{\mu_0 I}{4\pi a}(\sin\phi_1 + \sin\phi_2) \Rightarrow B = \frac{\mu_0 I}{4\pi a}(\sin 90° + \sin 90°)$$

$$\Rightarrow B = \frac{\mu_0 I}{4\pi a}(1+1) = \frac{\mu_0 I}{4\pi a}(2) \Rightarrow B_\infty = \frac{\mu_0 I}{2\pi a}$$

### Illustration 15:

A current of 20 A is established in a long wire along positive z-direction. The magnetic field B at the point (1m, 0, 0) is :-

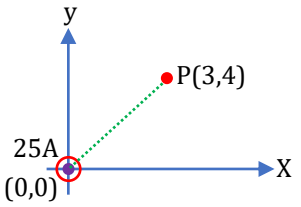
### Solution:

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 20}{2\pi(1)} = 2 \times 10^{-7} \times 20 = 4 \times 10^{-6}\ T = 4\mu T$$

direction of $\vec{B} \Rightarrow i\vec{d\ell} \times \vec{r} = \hat{k} \times \hat{i} = +\hat{j}$

### Illustration 16:

Find $\vec{B}$ at P.

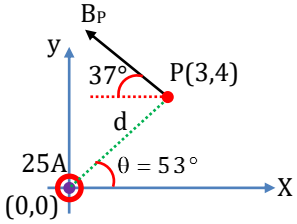



### Solution:

$$B_P = \frac{\mu_0 I}{2\pi d}$$

$$d = \sqrt{3^2 + 4^2} = 5$$

$$B_P = \frac{4\pi \times 10^{-7} \times 25}{2\pi \times 5} = 10 \times 10^{-7}\ T$$




In vector form,

$$\vec{B}_P = -B_p \cos 37\hat{i} + B_p \sin 37\hat{j}$$

$$= (-8 \times 10^{-7}\hat{i} + 6 \times 10^{-7}\hat{j})T$$

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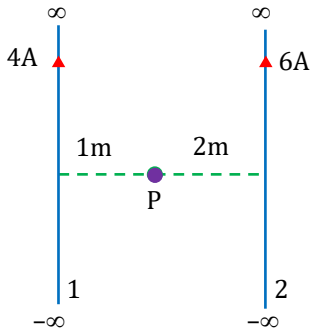
**Illustration 17:**

Find magnetic field at point P.




**Solution:**

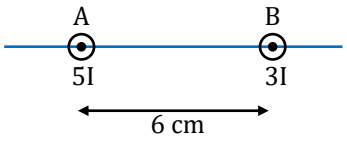
$$B_{1P} = \frac{\mu_0 I}{2\pi d}\otimes = \frac{\mu_0(4)}{2\pi(1)} \Rightarrow 8\times10^{-7}\,T\otimes$$

$$B_{2P} = \frac{\mu_0 I}{2\pi d}\odot = \frac{\mu_0(6)}{2\pi(2)} \Rightarrow 6\times10^{-7}\,T\odot$$

$$B_P = B_{1P} + B_{2P} = 2\times10^{-7}\,T\otimes$$

**Illustration 18:**

The position of point from wire 'B', where net magnetic field is zero due to following current distribution.



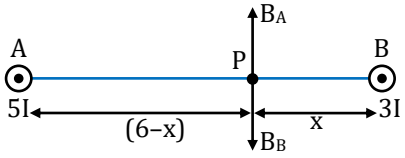

**Solution:**

Suppose magnetic field at the distance x from 'B' is zero so at that point magnetic field due to wires A and B will be equal in magnitude.

$B_A = B_B$

$$\frac{\mu_0(5I)}{2\pi(6-x)} = \frac{\mu_0(3I)}{2\pi x}$$

$$\frac{5}{(6-x)} = \frac{3}{x}\ ;\ x = \frac{9}{4}\,cm$$




So, $\frac{9}{4}$cm from B in left side magnetic field will be zero.

**Illustration 19:**

Same current i is flowing in three infinitely long wires along positive x, y and z directions. The magnetic field at a point (0, 0, –a) would be

**Solution:**

Point (0, 0, –a) lies on z-axis, therefore magnetic field due to current along z-axis is zero and due to rest two wires is $\frac{\mu_0 I}{2\pi a}$ in mutually perpendicular directions along positive y direction and negative x direction.

$$\therefore\ \vec{B} = \frac{\mu_0}{2\pi}\frac{i}{a}(\hat{j}-\hat{i})$$

## Magnetic Field due to Semi Infinite Straight Wire

**Case -1**

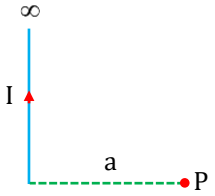
Here,

$\phi_1 \to 90°$

$\phi_2 = 0°$

$$B = \frac{\mu_0 I}{4\pi a}(\sin\phi_1 + \sin\phi_2)$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi a}(\sin 90° + \sin 0°) \Rightarrow B = \frac{\mu_0 I}{4\pi a}\otimes$$

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**Case -2**

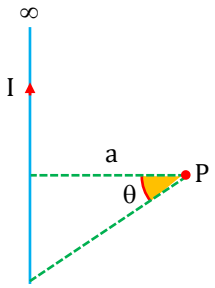
Here,

$\phi_1 \to 90°$

$\phi_2 = \theta$

$$B = \frac{\mu_0 I}{4\pi a}(\sin\phi_1 + \sin\phi_2)$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi a}(\sin 90° + \sin\theta) \Rightarrow B = \frac{\mu_0 I}{4\pi a}(1+\sin\theta)\otimes$$




**Case -3**

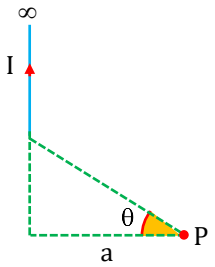
Here,

$\phi_1 \to 90°$

$\phi_2 = -\theta$

$$B = \frac{\mu_0 I}{4\pi a}(\sin\phi_1 + \sin\phi_2)$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi a}(\sin 90° + \sin(-\theta)) \Rightarrow B = \frac{\mu_0 I}{4\pi a}(1-\sin\theta)\otimes$$




**Case -4**

here,

$\phi_1 = -\theta$

$\phi_2 \to 90°$

$$B = \frac{\mu_0 I}{4\pi a}(\sin\phi_1 + \sin\phi_2)$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi a}(\sin(-\theta) + \sin 90°) \Rightarrow B = \frac{\mu_0 I}{4\pi a}(1-\sin\theta)\otimes$$

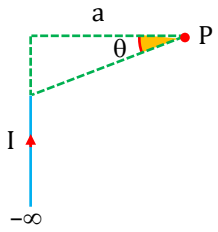



**Illustration 20:**

If point 'P' lies out-side the line of wire then magnetic field at point 'P' will be :

**Solution:**

Here $\phi_1 = (90° - \alpha_1)$ $\phi_2 = (90° - \alpha_2)$

$$B_P = \frac{\mu_0 I}{4\pi d}[\sin(90-\alpha_1) - \sin(90-\alpha_2)] = \frac{\mu_0 I}{4\pi d}(\cos\alpha_1 - \cos\alpha_2)$$

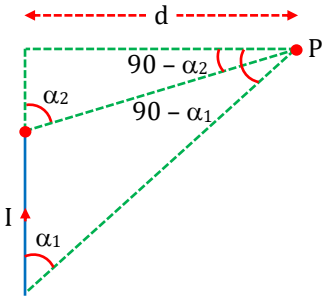
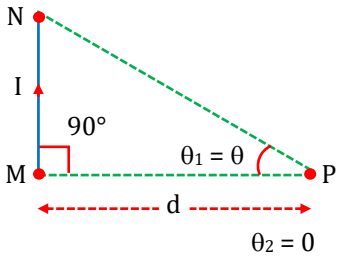



**Illustration 21:**

Find magnetic field due to finite length wire at point 'P'.




**Solution:**

$$B_P = \frac{\mu_0 I}{4\pi d}[\sin\theta + \sin 0°]; B_P = \frac{\mu_0 I}{4\pi d}\sin\theta$$

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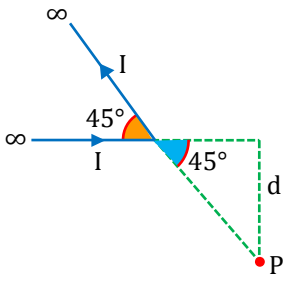
**Illustration 22:**

Find net magnetic field at point P.




**Solution:**

$$B = \frac{\mu_0 I}{4\pi d}(1-\sin\theta)$$

$$= \frac{\mu_0 I}{4\pi d}\left(1-\frac{1}{\sqrt{2}}\right)\otimes$$

## BEGINNER'S BOX-1

**1.** A proton moves along horizontal line and towards observer, the pattern of concentric circular field lines of magnetic field which produced due to its motion :-

(1) ACW, In horizontal plane (2) ACW, In vertical plane

(3) CW, In horizontal plane (4) CW, In vertical plane

**2.** An electron moves along vertical line and towards observer, the pattern of concentric circular field lines of magnetic field which is produced due to its motion :-

(1) ACW, In horizontal plane (2) ACW, In vertical plane

(3) CW, In horizontal plane (4) CW, In vertical plane

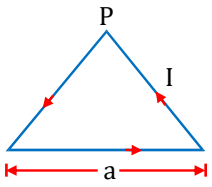
**3.** A current is flowing in electricity line towards north, the direction of magnetic field at a point which is just below the line :-

(1) towards north (2) towards south (3) towards east (4) towards west

**4.** A current is flowing in a vertical wire in downward direction, the direction of magnetic field of a point which is just right side to the wire :-

(1) towards north (2) towards south (3) towards east (4) towards west

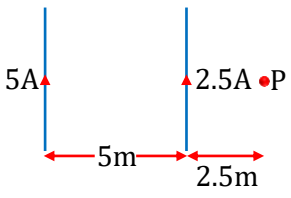
**5.** An equilateral triangle of side 'a' carries a current 'I'. Magnetic field at point 'P' which is vertex of triangle. :-




(1) $\frac{\mu_0 I}{2\sqrt{3}\pi a}\odot$ (2) $\frac{\mu_0 I}{2\sqrt{3}\pi a}\otimes$

(3) $\frac{9}{2}\left(\frac{\mu_0 I}{\pi a}\right)\odot$ (4) $\frac{9}{2}\left(\frac{\mu_0 I}{\pi a}\right)\otimes$

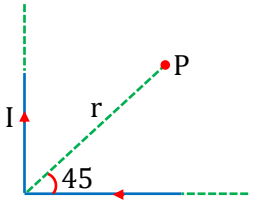
**6.** Magnetic field at point 'P' due to both infinite long current carrying wires is :-




(1) $\frac{\mu_0}{2\pi}\otimes$ (2) $\frac{5\mu_0}{6\pi}\otimes$

(3) $\frac{5\mu_0}{6\pi}\odot$ (4) $\frac{\mu_0}{2\pi}\odot$

**7.** Magnetic field at point 'P' due to given current distribution is :-




(1) $\frac{\mu_0 I}{4\pi r}\left(1+\sqrt{2}\right)\odot$ (2) $\frac{\mu_0 I}{2\pi r}\left(1+\sqrt{2}\right)\odot$

(3) $\frac{\mu_0 I}{4\pi r}\left(1+\sqrt{2}\right)\otimes$ (4) $\frac{\mu_0 I}{2\pi r}\left(1+\sqrt{2}\right)\otimes$

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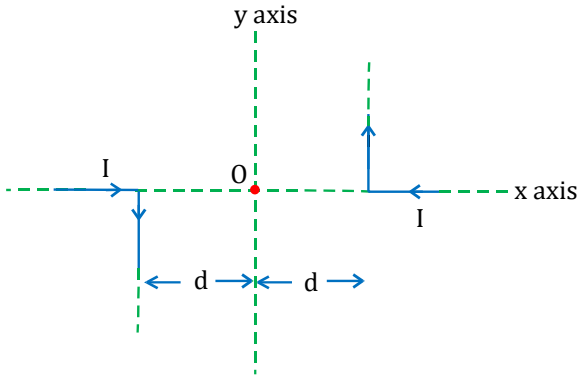
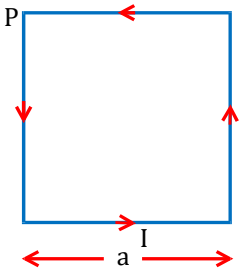
**8.** Magnetic field at origin 'O' due to given current distribution is :-




(1) $\frac{\mu_0 I}{2\pi d}\odot$ (2) $\frac{\mu_0 I}{2\pi d}\otimes$

(3) $\frac{\mu_0 I}{4\pi d}\odot$ (4) $\frac{\mu_0 I}{4\pi d}\otimes$

**9.** A square loop of side 'a' is made by a current carrying wire. Magnetic field at its vertex 'P' is :-



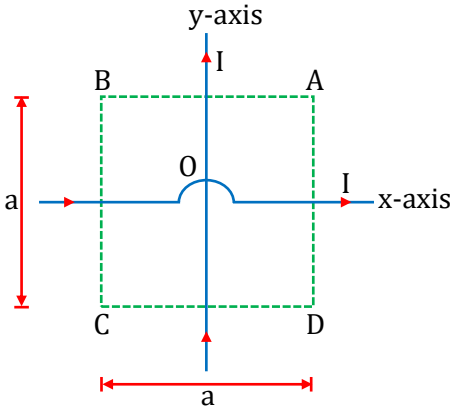

(1) $\frac{\mu_0 I}{2\sqrt{2}\pi a}\odot$ (2) $\frac{\mu_0 I}{2\sqrt{2}\pi a}\otimes$

(3) $2\sqrt{2}\frac{\mu_0 I}{\pi a}\otimes$ (4) $2\sqrt{2}\frac{\mu_0 I}{\pi a}\odot$

**10.** Two long parallel wires carry i and 2i current in same direction. Magnetic field just between the wires is 'B'. If 2i current is switched off then magnetic field at the same point is :-

(1) 2B (2) B (3) B/2 (4) $\sqrt{2}$ B

**11.** Two infinite length wires carry equal current and placed along x and y axis respectively. At which points the resultant magnetic field is zero?




(1) A, B (2) B, D (3) A, C (4) C, D

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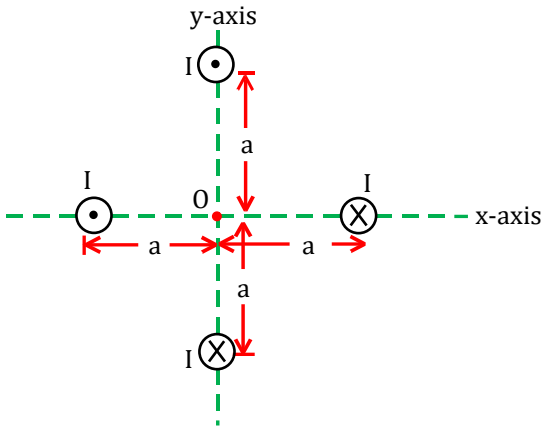
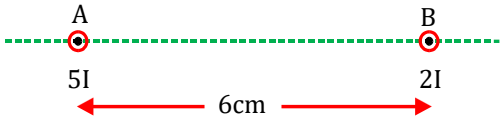
**12.** For given current distribution, each infinite length wire produces magnetic field 'B' at origin then resultant magnetic field at origin 'O' is :-




(1) 4B (2) $\sqrt{2}$ B (3) $2\sqrt{2}$ B (4) zero

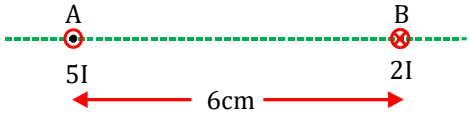
**13.** The position of point from wire 'B', where net magnetic field is zero due to following current distribution.

(a)




(1) 6/7 cm (2) 12/7 cm
(3) 18/17 cm (4) 16/7 cm

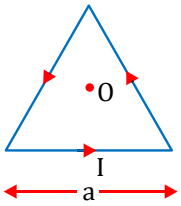
(b)




(1) 4 cm (2) 2 cm
(3) 8 cm (4) 12 cm

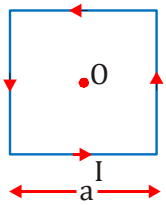
**14.** Magnetic field at the centre of various **regular polygons,** which are formed by current carrying wires.

(a) Equilateral triangle :



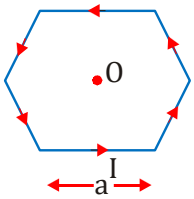

(1) $\frac{\mu_0 I}{2\sqrt{3}\pi a}$ (2) $\frac{27\mu_0 I}{2\pi a}$
(3) $\frac{9\mu_0 I}{2\pi a}$ (4) $\sqrt{3}\frac{\mu_0 I}{\pi a}$

(b) Square :




(1) $\frac{2\sqrt{2}\mu_0 I}{\pi a}$ (2) $\frac{\mu_0 I}{2\sqrt{2}\pi a}$
(3) $8\sqrt{2}\frac{\mu_0 I}{\pi a}$ (4) $\sqrt{2}\frac{\mu_o I}{\pi a}$

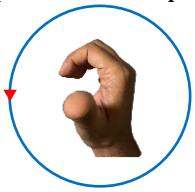
(c) Regular hexagon



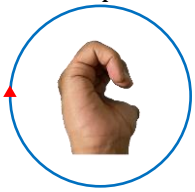

(1) $\frac{6\mu_o I}{\pi a}$ (2) $\frac{\sqrt{3}\mu_0 I}{\pi a}$
(3) $6\sqrt{3}\frac{\mu_0 I}{\pi a}$ (4) $\frac{\mu_o I}{\sqrt{3}\pi a}$

## Magnetic Field at the centre of Circular loop

**Direction:** Magnetic field is Perpendicular to plane of circular loop



Anti-
MF : Outwards



Clockwise
MF : Inwards

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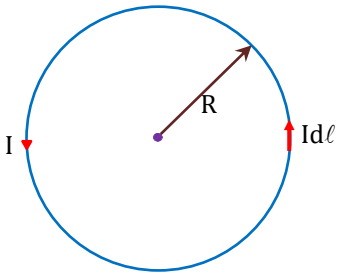
**Magnitude:**

M.F. at the center due to a current element

$$dB = \frac{\mu_0}{4\pi}\frac{Id\ell}{R^2}$$




Net M.F. at the center

$$B_0 = \frac{\mu_0}{4\pi}\frac{I}{R^2}\int d\ell = \frac{\mu_0}{4\pi}\frac{I}{R^2}(2\pi R)$$

If number of loops is not given, take single loop.

For single loop : - $B_0 = \frac{\mu_0 I}{2\pi}$

For N loops : - $B_0 = \frac{\mu_0 NI}{2R}$

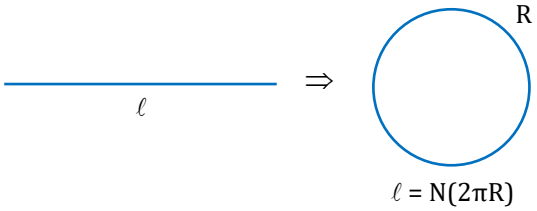
### Illustration 23:

Circular loop is made by a wire of length 7.5m. If current 5A is flowing in the loop, then find magnetic field at centre.

### Solution:

$$2\pi R = 7.5 \Rightarrow R = \frac{7.5}{2\pi} \Rightarrow B_C = \frac{\mu_0 I}{2R} \Rightarrow \frac{4\pi\times10^{-7}\times5}{2\times7.5/2\pi} = \frac{40\pi^2\times10^{-7}}{15}\,T$$

**Note :** If a single wire of constant length l is bent in form of N circular loops,




Then, M.F. at the center due to current I flowing in the loops will be

$$B_0 = \frac{\mu_0 NI}{2R} = \frac{\mu_0 NI}{2}\times\frac{1}{R} = \frac{\mu_0 NI}{2}\times\frac{2\pi N}{\ell} \Rightarrow B \propto N^2$$

### Illustration 24:

A circular ring of 2 turns is made by a conducting wire. The magnetic field at the center is B. Now, this same wire is bent to form circular ring of 4 turns, find magnitude of new magnetic field at the center (current is same).

### Solution:

$$B_1 \propto (N_1)^2 = B \propto 2^2 \Rightarrow B_2 \propto (N_2)^2 = B_2 \propto 4^2$$

From above equation

$$\frac{B_1}{B_2} = \frac{2^2}{4^2} \Rightarrow B_2 = 4B_1 = 4B$$

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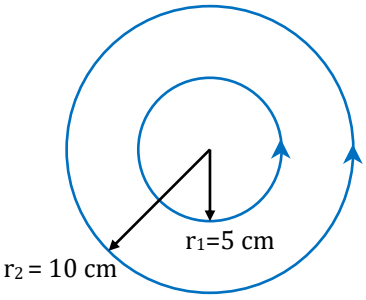
**Illustration 25:**

Two concentric circular coils of radius 5 cm and 10 cm are placed in same plane. If current 0.2 A is flowing in both coils in same direction, then find magnetic field at the centre.




**Solution:**

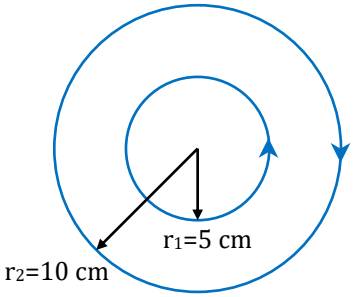
$$B_1 = \frac{\mu_0 I}{2r_1}\odot \Rightarrow B_2 = \frac{\mu_0 I}{2r_2}\odot$$

Net magnetic field at the centre ($B_0$) = $B_1 + B_2 = \frac{\mu_0 I}{2}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)$

$$B_0 = \frac{4\pi\times10^{-7}\times0.2}{2}\left(\frac{1}{0.5}+\frac{1}{0.1}\right) \Rightarrow B_0 = 4.8\pi\times10^{-7}\odot T$$

**Illustration 26:**

Two concentric circular coils of radius 5 cm and 10 cm are placed in same plane. If current 0.2 A is flowing in both coils in opposite direction, then find magnetic field at the centre.




**Solution:**

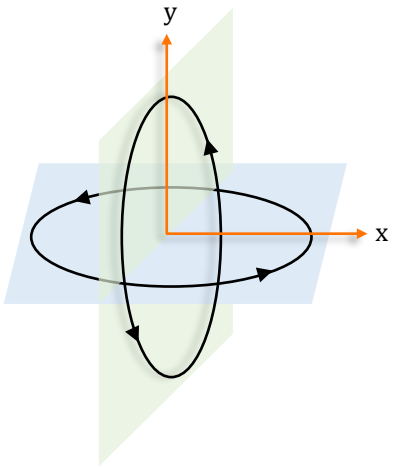
$$B_1 = \frac{\mu_0 I}{2r_1}\odot\ (+) \Rightarrow B_2 = \frac{\mu_0 I}{2r_2}\otimes\ (-)$$

Net magnetic field at the centre ($B_0$) = $B_1 + B_2 = \frac{\mu_0 I}{2}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$

$$B_0 = \frac{4\pi\times10^{-7}\times0.2}{2}\left(\frac{1}{0.05}-\frac{1}{0.1}\right) \Rightarrow B_0 = 4\pi\times10^{-7}\otimes T$$

**Illustration 27:**

Two symmetrical current carrying rings are placed $\perp$ to each other with common centre. If magnetic field at the centre due to one coil is B, then find net magnetic field.




**Solution:**

$$B_H = \frac{\mu_0 I}{2R} = B\hat{j} \Rightarrow B_V = \frac{\mu_0 I}{2R} = B\hat{i}$$

$$\Rightarrow B_{net} = \sqrt{B_H^2 + B_V^2} \Rightarrow B\sqrt{2}$$

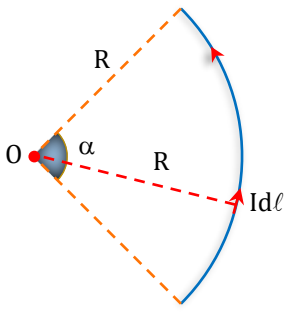
## Magnetic Field due to Arc

**Magnetic Field due to Circular Arc at the centre**

As per BSL,

Magnetic field due to current element $Id\ell$ at the centre of arc

$$dB = \frac{\mu_0}{4\pi}\frac{Id\ell}{R^2}$$




Due to complete arc, M.E. will be –

$$B = \frac{\mu_0}{4\pi}\frac{1}{R^2}\int d\ell \Rightarrow B = \frac{\mu_0}{4\pi}\frac{I}{R^2}(R\alpha) \Rightarrow B = \frac{\mu_0}{4\pi}\frac{I\alpha}{R}$$

**Note:** $\alpha$ must be in radian only

$$\theta^\circ\times\frac{\pi}{180^\circ}\rightarrow\theta \text{ rad}$$

Direction: by right hand thumb rule

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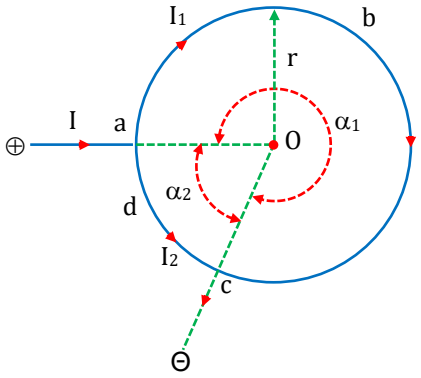
**(i) Wire of circuit loop is uniform:**




Magnetic field at the centre of arc abc and adc is

$$\left.\begin{aligned} B_{abc} &= \frac{\mu_0 I_1 \alpha_1}{4\pi r} \\ B_{adc} &= \frac{\mu_0 I_2 \alpha_2}{4\pi r} \end{aligned}\right\} \Rightarrow \frac{B_{abc}}{B_{adc}} = \frac{I_1\alpha_1}{I_2\alpha_2} \qquad ...(1)$$

$$\because \text{angle} = \frac{\text{arc length}}{\text{Radius}} \Rightarrow \frac{\alpha_1}{\alpha_2} = \frac{\ell_1}{\ell_2} \qquad ...(2)$$

and $$\left.\begin{aligned} V = IR \Rightarrow I \propto \frac{1}{R} \\ R = \frac{\rho\ell}{A} \Rightarrow R \propto \ell \end{aligned}\right\} \Rightarrow \frac{I_1}{I_2} = \frac{\ell_2}{\ell_1} \qquad ...(3)$$

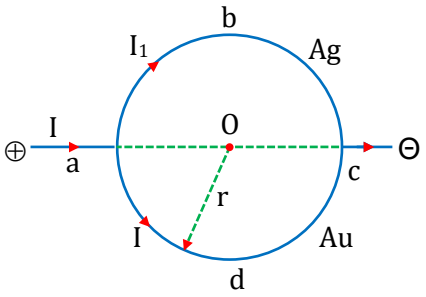
put the value of (2) & (3) in (1)

$$\frac{B_{abc}}{B_{adc}} = \left(\frac{\ell_2}{\ell_1}\right)\left(\frac{\ell_1}{\ell_2}\right) \Rightarrow \boxed{\frac{B_{abc}}{B_{adc}} = \frac{1}{1}}$$

- Magnetic field produced by both the arc is at the centre of the circuit loop is equal in magnitude and opposite in direction, So $(B_0)_{net} = 0$ (always and it is free from angle of connection of terminals)

**(ii) Wire of circuit loop is non-uniform:**

**Case I :**




Thickness ⟶ Same, Material ⟶ different: -

Magnetic field at the centre of arc abc and adc is –

$$B_{abc} = \frac{\mu_0 I_1}{4r}\otimes, B_{adc} = \frac{\mu_0 I_2}{4r}\odot$$

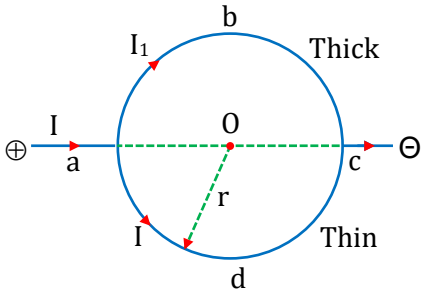
Both arc connected in parallel with source, so V = IR = constant

$$I \propto \frac{1}{R} \Rightarrow R_{A_u} > R_{A_g} \Rightarrow I_{A_u} < I_{A_g}$$

Net magnetic field at centre

$$(B_0)_{net} = \frac{\mu_0}{4r}(I_1 - I_2)\otimes \text{ (where } I_1 > I_2)$$

**Case II** :




Thickness ⟶ different, Material ⟶ same :–

Magnetic field at the centre of arc abc and adc

$$B_{abc} = \frac{\mu_0 I_1}{4r}\otimes, B_{adc} = \frac{\mu_0 I_2}{4r}\odot$$

Both arc are connected in parallel with source, so

V = IR = constant and resistance of wire $R = \frac{\rho\ell}{A}$

$$I \propto \frac{1}{R} \qquad ...(1) \qquad R \propto \frac{1}{A} \qquad ...(2)$$

From (1) & (2) $I \propto A \Rightarrow I_{thick} > I_{thin}$

Net magnetic field at centre

$$(B_0)_{net} = \frac{\mu_0}{4r}(I_1 - I_2)\otimes \text{ (where } I_1 > I_2)$$

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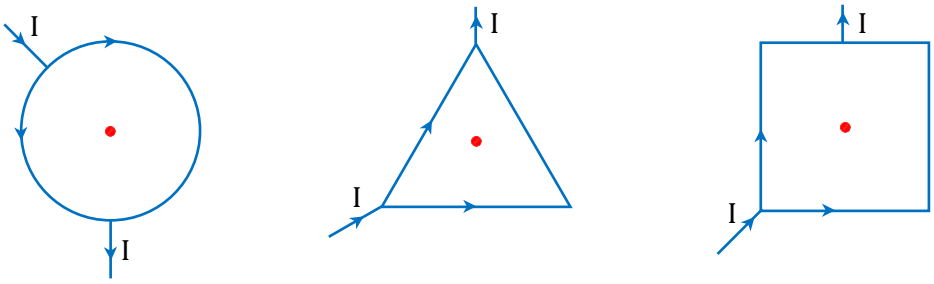
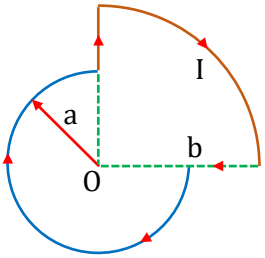
**Note:** When current divides in any symmetrical planar loop made with uniform wire, then magnetic field at the centre due to this loop is ZERO.




### Illustration 28:

The magnetic induction at the centre O is?




### Solution:

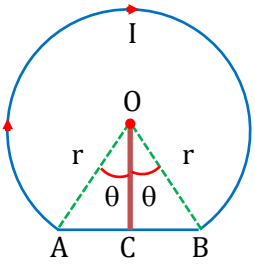
$$B=\frac{3}{4}\left[\frac{\mu_0 I}{2a}\right]\otimes+\frac{1}{4}\left[\frac{\mu_0 I}{2b}\right]\otimes$$

$$B=\left[\frac{3\mu_0 I}{8a}+\frac{\mu_0 I}{8b}\right]\otimes$$

### Illustration 29:

A wire is bent in the form of a circular arc with a straight portion AB. Magnetic induction at O when current I flowing in the wire, is?

### Solution:




$$B_{AB}=\frac{\mu_0 I}{4\pi(OC)}[2\sin\theta]$$

But $OC=r\cos\theta$ or $B_{AB}=\frac{\mu_0 I}{2\pi r}\tan\theta$

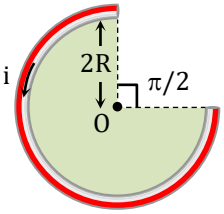
Magnetic field due to circular portion,

$$B_{AB}=\frac{\mu_0 I}{2r}\left(\frac{2\pi-2\theta}{2\pi}\right)=\frac{\mu_0 I}{2\pi r}(\pi-\theta)$$

Total magnetic field = $\frac{\mu_0 I}{2\pi r}\tan\theta+\frac{\mu_0 I}{2\pi r}(\pi-\theta)=\frac{\mu_0 I}{2\pi r}[\tan\theta+\pi-\theta]$

### Illustration 30:

A current i ampere flows in a circular arc of wire whose radius is 2R, which subtend an angle $3\pi/2$ radian at its centre. The magnetic induction B at the centre is

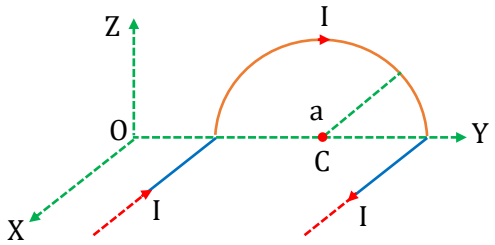



### Solution:

$$B=\frac{\mu_0}{4\pi}\frac{(2\pi-\theta)i}{R'}=\frac{\mu_0}{4\pi}\frac{\left(2\pi-\frac{\pi}{2}\right)\times i}{2R}=\frac{3\mu_0 i}{16R}$$

### Illustration 31:

A long wire bent as shown in the figure carries current I. If the radius of the semi-circular portion is "a" then find the magnetic induction at the centre C.




### Solution:

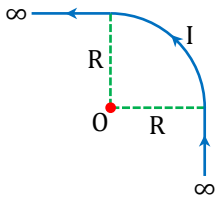
Due to semi-circular part $\vec{B}_1 = \frac{\mu_0 I}{4a}\left(-\hat{i}\right)$

Due to parallel parts of currents $\vec{B}_2 = 2\times\frac{\mu_0 I}{4\pi a}(-\hat{k}), \vec{B}_{net} = \vec{B}_C = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{4a}(-\hat{i}) + \frac{\mu_0 I}{2\pi a}(-\hat{k})$

Magnitude of resultant field $B_{net} = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0 I}{4\pi a}\sqrt{\pi^2 + 4}$

### Illustration 32:

Find magnetic field at O.



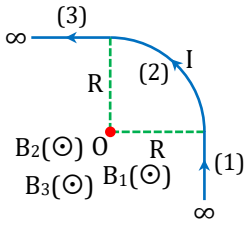

### Solution:

$B_{net} = B_1 + B_2 + B_3$

$$B_{net} = \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{4\pi R}\left(\frac{\pi}{2}\right) + \frac{\mu_0 I}{4\pi R}$$

$$= \frac{\mu_0 I}{2\pi R} + \frac{\mu_0 I}{8R}$$

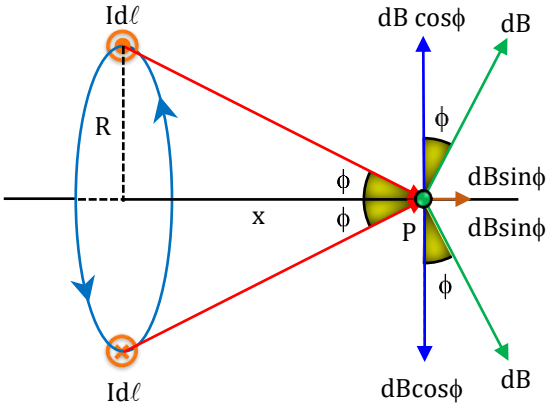
$$= \frac{\mu_0 I}{2R}\left[\frac{1}{\pi} + \frac{1}{4}\right]\odot$$




### Magnetic Field at the axis of Circular loop

**Magnetic Field on the Axis of Current Carrying circular Loop**

**Direction:** Component of magnetic field perpendicular to axis of ring will be zero because the components of magnetic field produced due to opposite current elements on the ring cancel each other. Net magnetic field due to ring at any point on the axis will be along the axis.



**Magnitude**

$$B_p = \int dB\sin\phi$$

$$B_p = \int \frac{\mu_0}{4\pi}\frac{Idl\sin(90°)}{r^2}\left(\frac{R}{r}\right)$$

$$B_P = \frac{\mu_0}{4\pi}\frac{IR}{r^3}\int d\ell = \frac{\mu_0}{4\pi}\frac{IR}{r^3}(2\pi R) = \frac{\mu_0}{2}\frac{IR^2}{r^3}$$

In terms of distance from center,

$$B_P = \frac{\mu_0}{2}\frac{IR^2}{\left(\sqrt{R^2+x^2}\right)^3} = \frac{\mu_0 IR^2}{2\left(R^2+x^2\right)^{2/3}}$$

To find magnetic field at the center of circular current loop($B_o$),
At center, (x) = 0

$$B_o = \frac{\mu_0 IR^2}{2\left(R^2+0\right)^{3/2}} = \frac{\mu_0 IR^2}{2R^3} = \frac{\mu_0 I}{2R}$$

Now,

$$B_P = \frac{\mu_0 IR^2}{2\left(R^2+x^2\right)^{3/2}} = \frac{\mu_0 IR^2}{2\left(R^2\right)^{3/2}\left(1+\frac{x^2}{R^2}\right)^{3/2}} = \frac{\mu_0 I}{2R\left(1+\frac{x^2}{R^2}\right)^{3/2}} = \frac{B_o}{\left(1+\frac{x^2}{R^2}\right)^{3/2}}$$

**Case 1 :** At very large distance from centre

$x >> R$

$x^2 + R^2 \approx x^2$

$$B \approx \frac{\mu_0 IR^2}{2\left(x^2\right)^{3/2}} \Rightarrow B \approx \frac{\mu_0 IR^2}{2x^3}$$

**Case 2 :** Near centre of the ring

$x << R$

$$B_P = \frac{B_0}{\left(1+\frac{x^2}{R^2}\right)^{3/2}} \Rightarrow B_P = B_0\left(1+\frac{x^2}{R^2}\right)^{-3/2}$$

$$B_P \approx B_0\left(1-\frac{3x^2}{2R^2}\right)$$

$$\frac{B_P}{B_0} = 1-\frac{3x^2}{2R^2}$$

$$\frac{B_0 - B_P}{B_0} = \frac{3x^2}{2R^2}$$

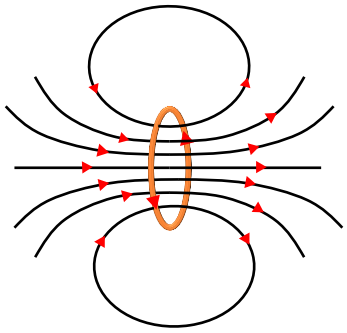
$$\frac{\Delta B}{B_0} = \frac{3x^2}{2R^2}$$ (Fractional change)

$$\Delta B = B_0\frac{3x^2}{2R^2}$$ (Valid upto only 5%)

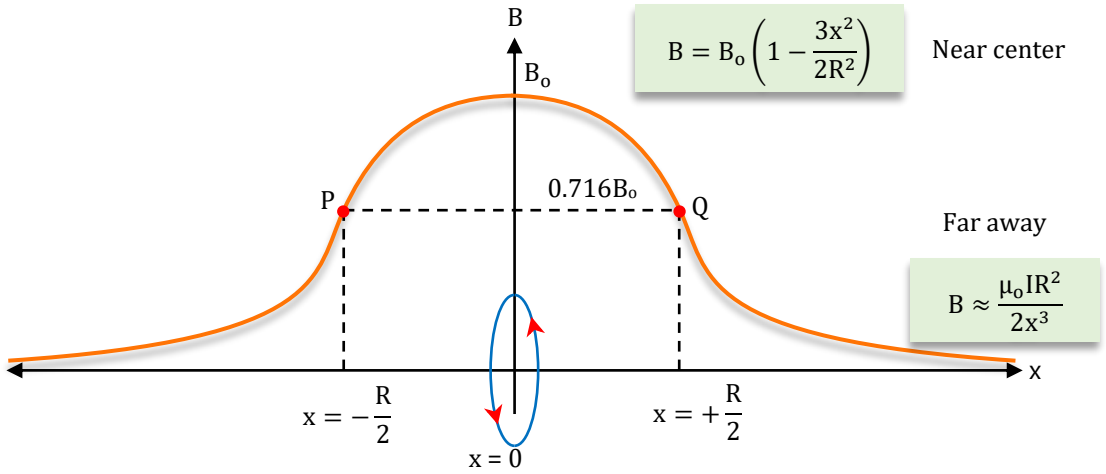
**Note:** 1. At x = 0, B is maximum ($B_0$)

2. At x = ∞, B = 0

3. On any point on the axis of current carrying coil, magnetic field is always along the axis (on both sides), direction can be found using RHTR.



## Graph : B vs x




At P and Q, graph changes its curvature

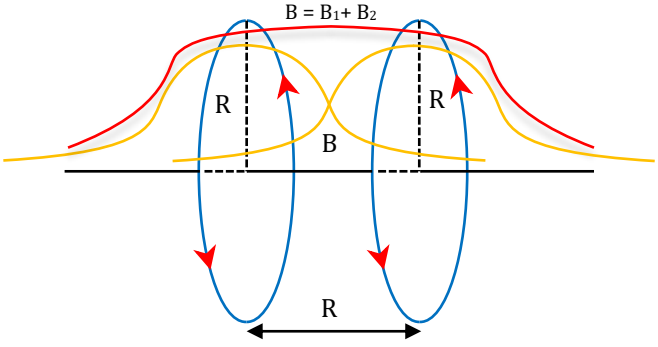
**P and Q are known as**

- Points of Inflection or
- Points of Zero curvature or
- Points of Curvature change.

## Helmholtz's Coil

Used to produce Uniform magnetic field (constant magnitude and direction) in a short range (in mm).

$$B \approx 0.72\frac{\mu_0 NI}{R}$$



**Conditions :**

1. Both coils must be coaxial
2. Same current in same sense
3. Equal number of turns and equal radius for both coils
4. Separation must be equal to R

### Illustration 33:

Find the magnetic field at axis (x = R/2) of circular current carrying loop in terms of $B_o$.

### Solution:

$$B_P = \frac{B_0}{\left(1+\frac{x^2}{R^2}\right)^{3/2}} = \frac{B_0}{\left(1+\frac{R^2}{4R^2}\right)^{3/2}} = \frac{B_0}{\left(\frac{5}{4}\right)^{3/2}} = \frac{8B_0}{5\sqrt{5}} = 0.716B_0$$

### Illustration 34:

Radius of a current carrying coil is 'R'. The ratio of magnetic field at a axial point which is 3R distance away from the centre of the coil to the magnetic field at the centre of the coil: -

### Solution:

$$B_x = \frac{B_0}{\left(1+\frac{x^2}{R^2}\right)^{3/2}}$$

$\because x = 3R$

$$\therefore B_x = \frac{B_0}{(1+9)^{3/2}} = \frac{B_0}{(10)^{3/2}} \Rightarrow \frac{B_x}{B_0} = \left(\frac{1}{10}\right)^{3/2} \Rightarrow B_x = B_0\left(\frac{1}{10}\right)^{3/2}$$

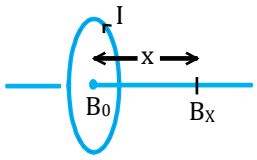
### Illustration 35:

Radius of current carrying coil is 'R'. If fractional decreases in field value with respect to centre of the coil for a nearby axial point is 3% then find axial position of that point.

### Solution:

Decrease in field value with respect to centre

$$\Delta B_X = B_0 - B_X = B_0 - B_0\left(1-\frac{3}{2}\frac{x^2}{R^2}\right) = B_0\left(\frac{3}{2}.\frac{x^2}{R^2}\right)$$




Fractional decrease in field value (Where x <<< R)

$$\frac{\Delta B_x}{B_0} = \frac{3}{2}\frac{x^2}{R^2} \text{ (limit} \le 5\%) \Rightarrow \frac{3}{100} = \frac{3}{2}\frac{x^2}{R^2} \Rightarrow x = \pm\frac{R}{\sqrt{50}}$$

### Illustration 36:

Magnetic fields at two points on the axis of a circular coil at a distance of 0.04m and 0.2m from the centre are in the ratio 8 : 1. The radius of the coil is

### Solution:

$$B = \frac{\mu_0}{4\pi} \times \frac{2\pi NiR^2}{(R^2+x^2)^{3/2}} \Rightarrow B \propto \frac{1}{(r^2+x^2)^{3/2}}$$

$$\Rightarrow \frac{8}{1} = \frac{(R^2+x_2^2)^{3/2}}{(R^2+x_1^2)^{3/2}} \Rightarrow \left(\frac{8}{1}\right)^{2/3} = \frac{R^2+0.04}{R^2+0.0016}$$

$$\Rightarrow \frac{4}{1} = \frac{R^2+0.04}{R^2+0.0016}$$ On solving R = 0.10 m

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## BEGINNER'S BOX-2

**1.** The magnetic field at the centre of a circular coil of radius r carrying current is $B_1$. The field at the centre of another coil of radius $\frac{r}{2}$ carrying same current is $B_2$ then ratio of $B_1/B_2$ is :-

(1) 1 : 2 (2) 2 : 1 (3) 1 : 1 (4) 4 : 1

**2.** A circular coil of one turn in formed by a 6.28m length wire, which carries a current of 3.14A. The magnetic field at the centre of coil is :-

(1) $1 \times 10^{-6}$ T (2) $4 \times 10^{-6}$ T (3) $0.5 \times 10^{-6}$ T (4) $2 \times 10^{-6}$ T

**3.** A length of wire carries a steady current, is bent first to form a plane circular coil of one turn, same length now bent more sharply to give three turns of smaller radius. Magnetic field becomes :-

(1) 3 times (2) 1/3 times (3) 9 times (4) unchange

**4.** Two concentric coplanner coils of equal turns have radii 10 cm and 30 cm respectively. Same current flowing in both the coils in same direction. Now direction of current is reversed in one coil then ratio of magnetic field at their common centre in two conditions respectively :-

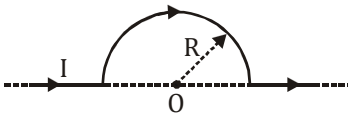
(1) 2 : 1 (2) 1 : 2 (3) 1 : 1 (4) 4 : 1

**5.** Two concentric coplanner coils of turns $n_1$ and $n_2$ have radii ratio 2 : 1 respectively. Equal current in both the coils flows in opposite direction. If net magnetic field is zero at their common centre then $n_1 : n_2$ is :-

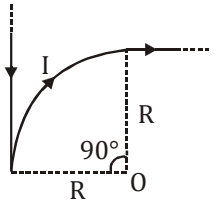
(1) 2 : 1 (2) 1 : 2 (3) 1 : 1 (4) 4 : 1

**6.** Find out magnetic field at point 'O' for the following current distributions.

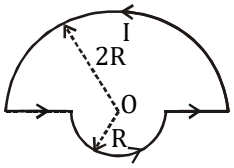
(i) (a)



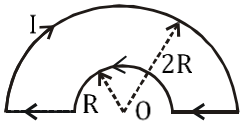

(b)



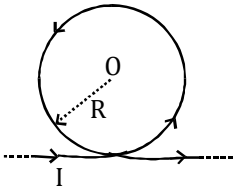

(ii) (a)



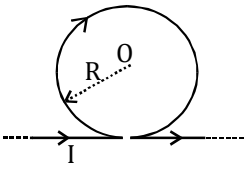

(b)



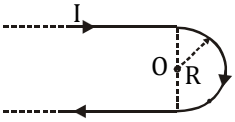

(iii) (a)



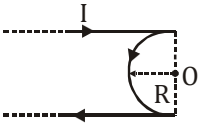

(b)



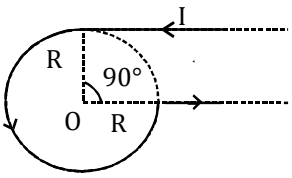

(iv) (a)



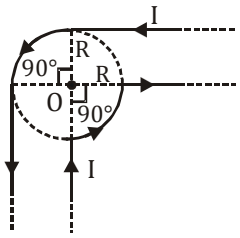

(b)




(v) (a)




(b)

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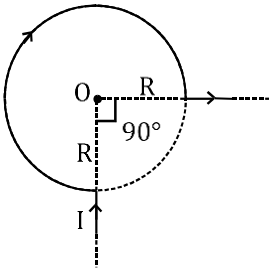
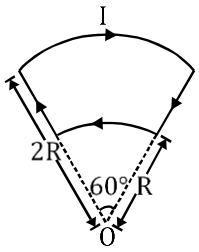
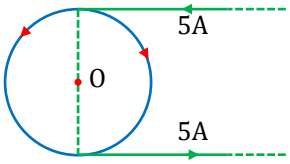
(vi) (a)




(b)

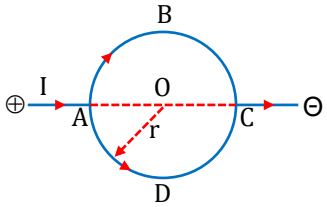



**7.** Magnetic field at point 'O' due to given current distribution. If 5A current is flowing in this system and the diameter of the loop is 10cm.




(1) $2 \times 10^{-5}$ T, $\otimes$ (2) $10^{-5}$T, $\odot$ (3) $10^{-5}$ T, $\otimes$ (4) $2 \times 10^{-5}$ T, $\odot$

**8.** Figure shows a circular loop with radius 'r'. The resistance of arc ABC is 5Ω and that of ADC is 10Ω. Magnetic field at the centre of the loop is :-

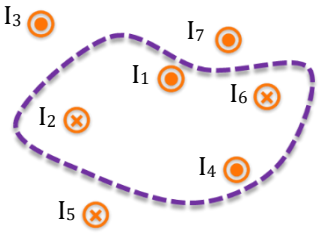



(1) $\frac{\mu_0 I}{6r}\otimes$ (2) $\frac{\mu_0 I}{12r}\otimes$ (3) $\frac{\mu_0 I}{12r}\odot$ (4) $\frac{\mu_0 I}{6r}\odot$

**9.** Radius of current carrying coil is 'R'. If fractional decreases in field value with respect to centre of the coil for a near by axial point is 1% then find axial position of that point.

## Ampere's Circuital Law and its basic applications

### Ampere Circuital Law




It gives another method to calculate the magnetic field due to given current distribution.

Line integral of magnetic field along any closed loop is equal to $\mu_0$ times the net current crossing the surface bounded by the loop.

Mathematically,

$$\oint \vec{B}\cdot\vec{d\ell} = \mu_0 I_{enc}$$

**Note:** 1. It is applicable only for steady/constant current

2. It can be applied for any distribution of current **but**,
3. It is applied for **symmetric distribution** only for calculation purpose
4. Any shape of loop is acceptable **but**,
5. $\vec{B}$ and $\vec{d\ell}$ should be **parallel or perpendicular** to each other.

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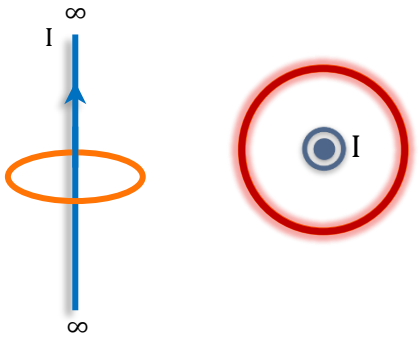
## Magnetic Field due to Long Thin wire

$\oint \vec{B}\cdot\vec{d\ell} = \mu_0 I_{enc}$

$B(2\pi r) = \mu_0 (I)$

$B = \frac{\mu_0 (I)}{2\pi r}$

$B = \frac{\mu_0 I}{2\pi r}$




## Magnetic Field due to Long Thick wire

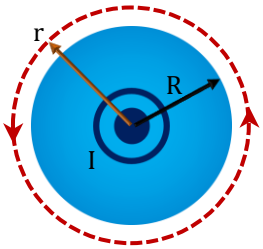
At a point outside wire

$\oint \vec{B}\cdot\vec{d\ell} = \mu_0 I_{enc}$

$B(2\pi r) = \mu_0 (I)$

$B = \frac{\mu_0 (I)}{2\pi r}$

$B_{out} = \frac{\mu_0 I}{2\pi r}$, $B_{sur} = \frac{\mu_0 I}{2\pi R}$



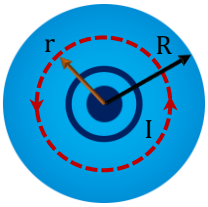

At a point inside wire

$\oint \vec{B}\cdot\vec{d\ell} = \mu_0 I_{enc}$

$B(2\pi r) = \mu_0 \left( \frac{I}{\pi R^2} \times \pi r^2 \right)$

$B = \frac{\mu_0 (Ir)}{2\pi R^2}$

$B_{in} = \frac{\mu_0 I}{2\pi R^2} r = \frac{\mu_0 J r}{2}$ (Current density, $J = \frac{I}{\pi R^2}$)




A : Outside the wire surface

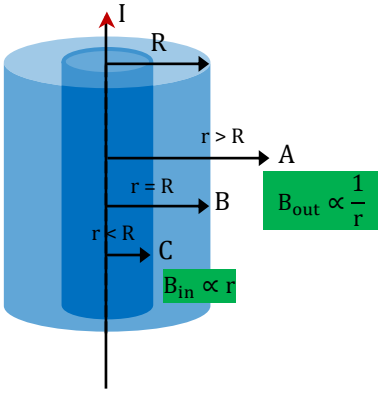
$B_{out} = \frac{\mu_0 I}{2\pi r}$, $B_{out} \propto \frac{1}{r}$

B : On the wire surface

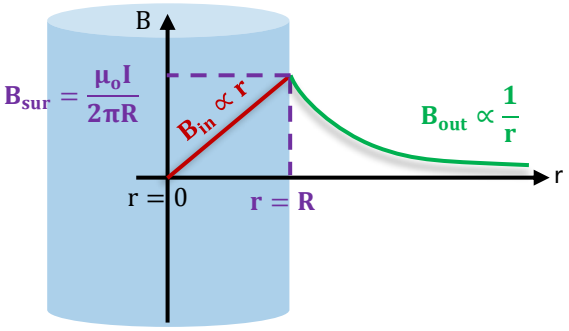
$B_{sur} = \frac{\mu_0 I}{2\pi R}$

C : Inside the wire surface

$B_{in} = \frac{\mu_0 I}{2\pi R^2} r$, $B_{in} \propto r$




**Graph**




Magnetic field is maximum at the surface of wire.

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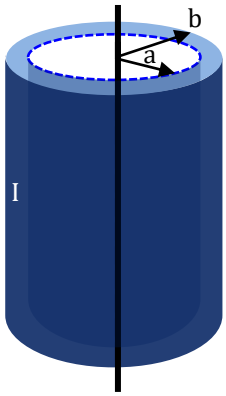
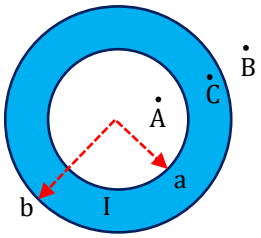
## Magnetic Field due to Long Hollow Cylindrical Conductor




At point A, r < a

$I_{enc} = 0 \Rightarrow B_{in} = 0$

At point B, r ≥ b

$I_{enc} = I \Rightarrow B_{out} = \frac{\mu_0 I}{2\pi r}$

At point C, b > r ≥ a

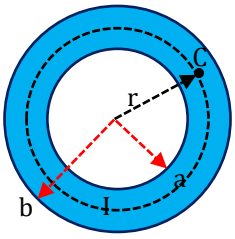
$I_{enc} = \frac{I}{\pi(b^2 - a^2)} \times \left(\pi(r^2 - a^2)\right)$

$I_{enc} = I\left(\frac{r^2 - a^2}{b^2 - a^2}\right)$

$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enc}$

$B(2\pi r) = \mu_o I_{enc}$

$B_{mid} = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2 - a^2}{b^2 - a^2}\right)$




### Illustration 37:

A current of $1/(2\pi)$ ampere is flowing in a long straight conductor. The line integral of magnetic induction around a closed path enclosing the current carrying conductor is.

### Solution:

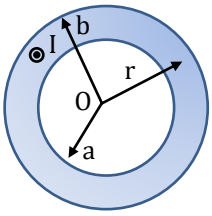
$\int \vec{B} \cdot d\vec{l} = \mu_0 I = 4\pi \times 10^{-7} \times \frac{1}{2\pi} = 2 \times 10^{-7}$ weber/metre

### Illustration 38:

A hollow cylindrical wire carries a current 5A, having inner and outer radii 'R' and 2R respectively. Magnetic field at a point which 3R/2 distance away from its axis is (R = 5m) : -

### Solution:

Field inside cross-section of conductor




$B_p = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2 - a^2}{b^2 - a^2}\right)$

[a=R, b=2R, r = 3R/2]

$B_p = \frac{\mu_0 I}{2\pi\left(\frac{3R}{2}\right)}\left[\frac{(3R/2)^2 - (R)^2}{(2R)^2 - (R)^2}\right] = \frac{5}{36} \cdot \frac{\mu_0 I}{\pi R} = \frac{5}{36} \cdot \frac{5 \times \mu_0}{5 \times \pi} = \frac{5}{9} \times 10^{-7}\,T$

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## BEGINNER'S BOX-3

**1.** A solid cylindrical wire of radius 'R' Carries a current 'I'. The ratio of magnetic fields at points which are located at R/2 and 2R distance away from the axis of the wire :-

(1) 1 : 1 (2) 1 : 2 (3) 2 : 1 (4) 1 : 4

**2.** A solid cylindrical wire of radius 'R' carries a current 'I'. The magnetic field is 5μT at a point, which is '2R' distance away from the axis of wire. Magnetic field at a point which is R/3 distance inside from the surface of the wire is :-

(1) $\frac{10}{3}\mu T$ (2) $\frac{20}{3}\mu T$ (3) $\frac{5}{3}\mu T$ (4) $\frac{40}{3}\mu T$

**3.** A hollow cylindrical wire carries a current I, having inner and outer radii 'R' and 2R respectively. Magnetic field at a point which 3R/2 distance away from its axis is :-

(1) $\frac{5\mu_0 I}{18\pi R}$ (2) $\frac{\mu_0 I}{36\pi R}$ (3) $\frac{5\mu_0 I}{36\pi R}$ (4) $\frac{5\mu_0 I}{9\pi R}$

**4.** A long, straight and solid metal wire of radius 2mm carries a current uniformly distributed over its circular cross section. The magnetic field at a distance 2 mm from its axis is B. Magnetic field at a distance 1 mm from the axis of the wire is :-

(1) B (2) 4B (3) 2B (4) B/2

**5.** A long straight wire (radius = 3.0 mm) carries a constant current distributed uniformly over a cross section perpendicular to the axis of the wire. If the current density is 100 $A/m^2$. The magnitudes of the magnetic field at (a) 2.0 mm from the axis of the wire and (b) 4.0 mm from the axis of the wire is :-

(1) $2\pi \times 10^{-8}$ T, $\frac{9\pi}{2}\times 10^{-8}$T (2) $4\pi \times 10^{-8}$ T, $\frac{\pi}{2}\times 10^{-8}$T

(3) $4\pi \times 10^{-8}$ T, $\frac{9\pi}{2}\times 10^{-8}$T (4) $\pi \times 10^{-4}$ T, $\frac{9\pi}{2}\times 10^{-8}$T

## Magnetic field due to Solenoid

**Solenoid**

- It consists of a conducting wire which is tightly wound over a cylindrical frame in the form of helix. All the adjacent turns are electrically insulated to each other. The magnetic field at a point on the axis of a solenoid can be obtained by superposition of field due to large number of identical circular turns having their centres on the axis of solenoid.
- M.F. is produced along axis of solenoid. Outside a long solenoid, M.F. is almost Zero.
- Direction of magnetic field $(\vec{B})$ inside a solenoid can be obtained by using **Right Hand Thumb Rule**.

(a)

(b)

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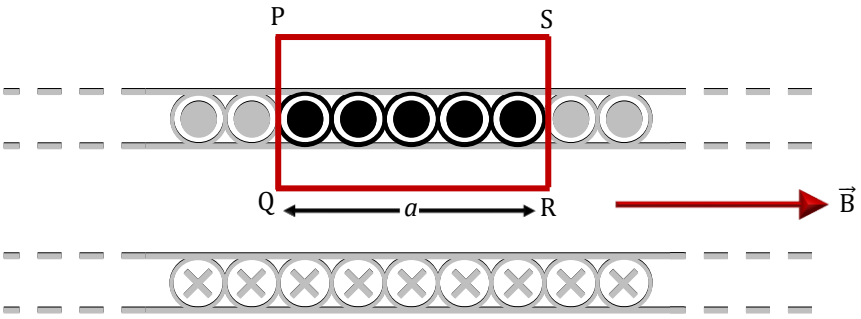
**Magnetic Field Inside a Long Solenoid**




For the loop PQRS, we have

$$\oint \vec{B}\cdot d\vec{\ell} = \int_{PQ} \vec{B}\cdot d\vec{\ell} + \int_{QR} \vec{B}\cdot d\vec{\ell} + \int_{RS} \vec{B}\cdot d\vec{\ell} + \int_{SP} \vec{B}\cdot d\vec{\ell}$$

Here, $\oint_{PQ} \vec{B}.d\vec{\ell} = \oint_{RS} \vec{B}.d\vec{\ell} = 0 \quad (\because \vec{B} \perp d\vec{\ell})$ and $\oint_{SP} \vec{B}.d\vec{\ell} = 0 \quad (\because \vec{B}$ outside the solenoid is negligible)

$$\int_{QR} \vec{B}\cdot d\vec{\ell} = B\int_{QR} d\ell = Ba$$

Now, $\oint \vec{B}\cdot d\vec{\ell} = Ba$

Current enclosed for loop PQRS, $\Sigma i = (n\times a)\times i$ where n = number of turns per unit length

i = current through wire of solenoid

From Ampere's Circuital Law for the loop PQRS, we have

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 \Sigma i$$

$$B\times a = \mu_0 (n\times a\times i)$$

$$B = \mu_0 ni$$

**General Parameters of Solenoid**

N : Number of turns

$\ell$ : Length of solenoid

n : Number of turns per unit length

Hence $n = \frac{N}{\ell}$

R : Radius of solenoid coil

$\mu_r$ : Relative permeability of core of solenoid

**Note:** For a long solenoid length of solenoid is very large as compared to radius of solenoid i.e. $\ell >> r$

**Magnetic Field due to Long Solenoid**

For a long solenoid, carrying current I

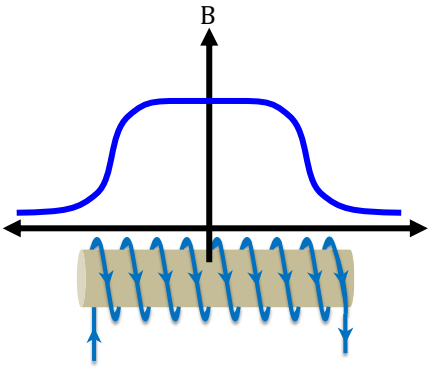
M.F. in the central region $B_0 = \mu_0 nI \qquad B_0 = \mu_0\left(\frac{N}{\ell}\right)I$

M.F. outside the solenoid B = 0

M.F. at the edges/end points $B = \frac{\mu_0 nI}{2}$

In presence of Core, $\mu \to \mu_0 \mu_r$

Where $\mu_r$ is the relative permeability of core material.

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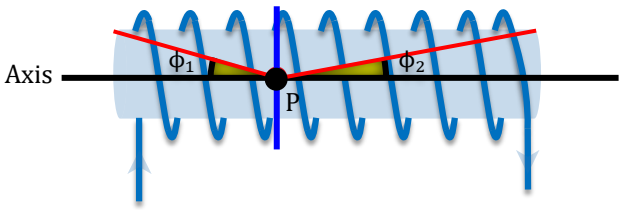
**Magnetic Field due to Finite Length Solenoid**




$$B_P = \frac{\mu_0 nI}{2}(\cos\phi_1 + \cos\phi_2)$$

## Note:

**Magnetic field outside a finite length solenoid is not zero.**

For long solenoid, in the middle region

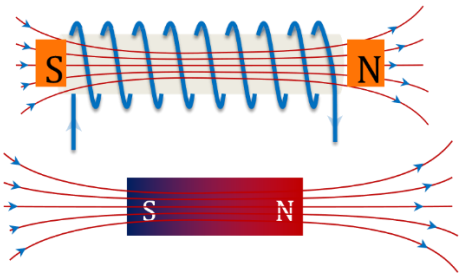
$$\phi_1, \phi_2 \to 0^o \quad \Rightarrow \quad B = \mu_0 nI\frac{(\cos 0^o + \cos 0^o)}{2} \quad \Rightarrow \quad B = \mu_0 nI$$

For long solenoid, in the end region

$$\phi_1, \phi_2 \to 0^o, 90^o \quad \Rightarrow \quad B = \mu_0 nI\frac{(\cos 0^o + \cos 90^o)}{2} \quad \Rightarrow \quad B = \frac{\mu_0 nI}{2}$$

## Key Note

Magnetic Field produced by a Solenoid resembles the Magnetic field of a Bar Magnet




So, a solenoid acts as a Magnetic Dipole and is used as Electromagnet in many devices.

## Illustration 39:

The length of solenoid is 0.1m and its diameter is very small. A wire is wound over it in two layers. The number of turns in inner layer is 50 and that of outer layer is 40. The strength of current flowing in two layers in opposite direction is 3A. Then find magnetic induction at the middle of the solenoid.

## Solution:

Direction of magnetic field due to both layers is opposite as direction of current is opposite, so

$$B_{net} = B_1 - B_2 = \mu_0 n_1 I_1 - \mu_0 n_2 I_2 = \mu_0 \frac{N_1}{\ell} I - \mu_0 \frac{N_2}{\ell} I \quad (\because I_1 = I_2 = I)$$

$$B_{net} = \frac{\mu_0 I}{\ell}(N_1 - N_2) = \frac{4\pi \times 10^{-7} \times 3}{0.1}(50 - 40) = 12\pi \times 10^{-5}\,T$$

## Illustration 40:

A closely wound, solenoid 80 cm long has 5 layers of winding of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0A. Estimate the magnetic field (a) Inside the solenoid (b) Axial end points of the solenoid

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**Solution:**

(a) Magnetic field inside the solenoid

$$B_{in} = \mu_0 nI = \mu_0 \frac{N}{\ell} I, \left(N = 400 \times 5 = 2000\right) = \frac{4\pi \times 10^{-7} \times 2000}{\left(80 \times 10^{-2}\right)} = \pi \times 10^{-3} T$$

(b) Magnetic field at axial end points of solenoid $B_{ends} = \frac{\mu_0 nI}{2} = \frac{\pi \times 10^{-3}}{2} = \frac{\pi}{2} \times 10^{-3} T$

**Illustration 41:**

A straight long solenoid produces magnetic field 'B' at its centre. If it is cut into two equal parts and same number of turns are wound on one part in double layer. Find magnetic field produced by new solenoid at its centre.

**Solution:**

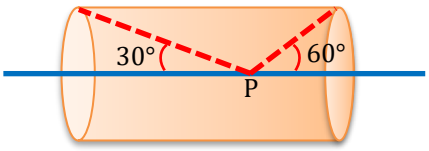
Magnetic field produced by a long solenoid is $B = \mu_0 nI$, where $n = N/\ell$

$\because$ Same number of turns wound over half length

$\therefore$ Magnetic field produced by new solenoid is $B' = \mu_0 \left(\frac{N}{\ell/2}\right) I = 2\left(\frac{\mu_0 NI}{\ell}\right) = 2B$

**Illustration 42:**

Find out magnetic field at axial point 'P' of solenoid shown in figure(where turn density 'n' and current through it is I)



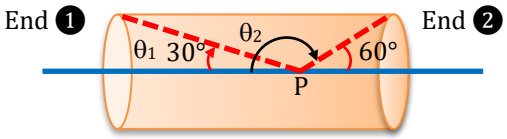

**Solution:**

Magnetic field at point 'P' due to finite length solenoid

$$B_P = \frac{\mu_0 nI}{2}\left(\cos\phi_1 + \cos\phi_2\right)$$

$$= \frac{\mu_0 nI}{2}\left[\cos 30° + \cos 60°\right]$$

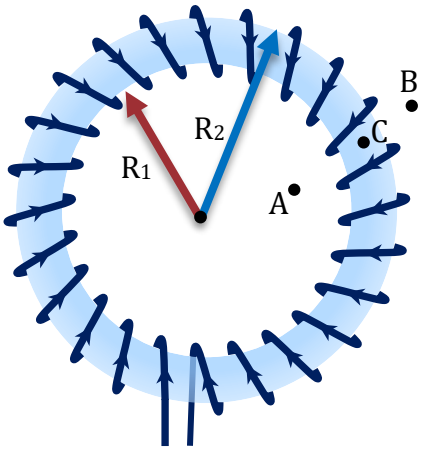
$$= \frac{\mu_0 nI}{2}\left[\frac{\sqrt{3}}{2} + \frac{1}{2}\right] = \frac{\mu_0 nI}{4}\left(\sqrt{3}+1\right)$$




**Magnetic field due to Toroid**

A toroid can be considered as a ring-shaped closed solenoid also called endless solenoid.

Toroid is a solenoid bent in ring shape.



Magnetic field inside a toroid by A.C.L. given as: –

Point A: Magnetic field in the empty space surrounded by toroid ($r < R_1$)

Since $I_{enc} = 0 \Rightarrow B = 0$

Point B: Magnetic field outside the toroid ($r > R_2$)

Since $I_{enc} = NI - NI = 0 \Rightarrow B = 0$

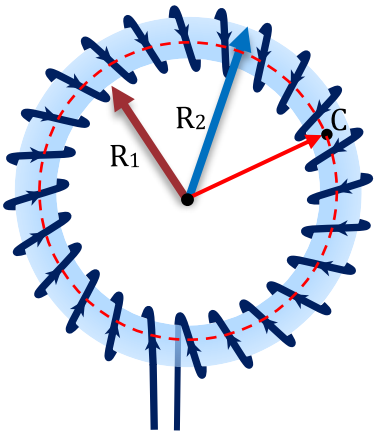
Point C: Inside the Toroid ($R_1 \le r \le R_2$)

$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enc}$$

$$B(2\pi r) = \mu_o (NI)$$

$$B = \mu_o \frac{N}{2\pi r} I$$

$$B = \mu_o n I$$




If r is not given then for magnetic field inside the toroid

$$n = \frac{N}{2\pi r_m}$$

$$\left[ R_m = \frac{R_1 + R_2}{2} = \text{mean radius of toroid} \right]$$

**Illustration 43:**

A toroid has a core (nonferromagnetic) of inner radius ($R_1$) 25 cm and outer radius ($R_2$) 26 cm around which 3500 turns of a wire are wound. If the current in the wire is 11 A, what is the magnetic field: (a) Outside the toroid (b) Inside the core of the toroid (c) In the empty space surrounded by the toroid.

**Solution:**

We are given that, mean radius of the toroid, i.e.,

$$r = \frac{25\text{cm} + 26\text{cm}}{2} = 25.5 \text{ cm} = 0.255 \text{ m}$$

total number of turns, N = 3500

current through the toroid, I = 11 A

Clearly number of turns per unit length of the toroid,

$$n = \frac{N}{2\pi r} = \frac{3500}{2\pi \times 0.255}$$

(a) Magnetic field outside the toroid is zero as it non-zero inside its core.

(b) Magnetic field inside the core of the toroid, i.e.,

$$B = \mu_0 n I = \left[ (4\pi \times 10^{-7}) \left( \frac{3500}{2\pi \times 0.255} \right) \times 11 \right] T = 3.0 \times 10^{-2} T$$

**Note :** Inside the toroid, $B \propto \frac{1}{r}$. The above value of B corresponds to mean radius 25.5 cm.

(c) In the empty space surrounded by the toroid, B = 0

**Key Note**

- Magnetic field of toroid is non-uniform because at each point direction of magnetic field is not same.
- It is not a Magnetic Dipole
- Magnetic field produced by a toroid is directed along its circular axis and constant in magnitude over its entire cross section.
- Magnetic field at the centre of toroid is always zero.

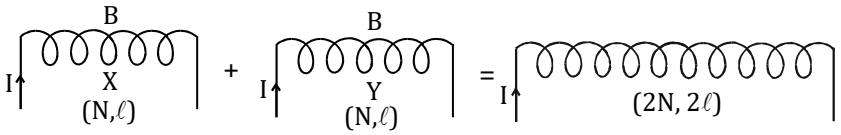
## BEGINNER'S BOX-4

**1.** State the following statements are true or false :

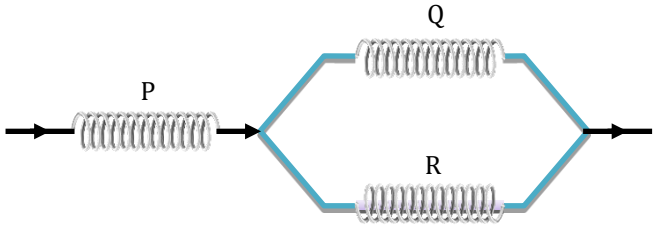
(a) The magnetic field inside straight solenoid of finite length is always less than $\mu_0 nI$.

(b) The magnetic field of an infinitely long ideal solenoid of radius 'R' carrying current I is constant inside and zero outside.

**2.** Two identical long solenoids X and Y carries equal current. The magnetic field produced at their axial mid point is B. If both the solenoids joined to each other according to figure. Find magnetic field at axial mid point of new solenoid.



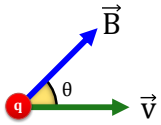

**3.** Three identical long solenoids P, Q and R connected to each other as shown in figure. If the magnetic field at the centre of P is 2.0 T, what would be the field at the centre of Q?




## Motion of Charge in Magnetic Field

### Magnetic Force on Moving Charge

Consider a charge q moving with velocity $\vec{v}$ in uniform magnetic field $\vec{B}$ such that $\vec{v}$ makes an angle $\theta$ with $\vec{B}$, then magnetic force experienced by the charge is given by




$\vec{F} = q(\vec{v} \times \vec{B})$

$F = qvB \sin\theta$

Important points

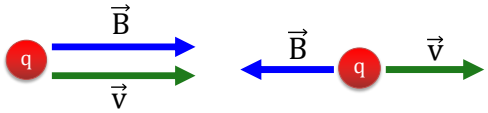
(1) Since magnetic force, $\vec{F} = q(\vec{v} \times \vec{B})$

$\vec{F} \perp \vec{v}$ $\quad \vec{F} \perp \vec{B}$

(2) If v = 0, then $F = qvB \sin\theta = 0$. So, there will be no magnetic force on stationary charge.

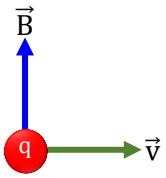
(3) Magnetic force depends on angle between $\vec{v}$ and $\vec{B}$

**Case-1:** $\theta = 0°$ or $180°$




$\Rightarrow F = 0$

**Case-2:** $\theta = 90°$




$\Rightarrow F_{max} = qvB$

($\theta$ can vary from 0° to 180°)

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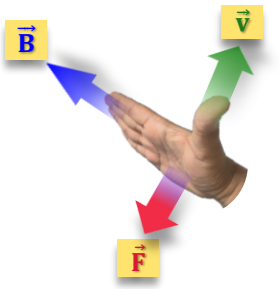
(4) Direction of force can be identified by right hand thumb rule or right hand palm rule.

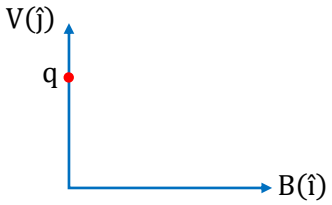
Right hand thumb rule : Point the fingers of right hand in the direction of $\vec{v}$, and curl them towards $\vec{B}$ then thumb will give the direction of $\vec{v} \times \vec{B}$

**Note:** For force on negative charge, just reverse the final direction.




### Illustration 44:

Find direction of & plane of motion.

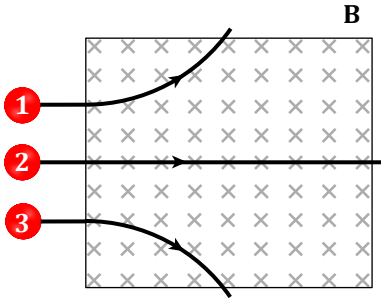



### Solution:

$\vec{F} = q\left(\vec{v} \times \vec{B}\right) \Rightarrow \hat{j} \times \hat{i} = \left(-\hat{k}\right) \Rightarrow \vec{F} \to \left(-\hat{k}\right)$ $\otimes$ y z plane

### Illustration 45:

From the path followed, identify nature of charge on the particle.




### Solution:

(1) Positive charge = According to Right-Hand Rule positive charge will move in circular upward direction.

(2) Since F = 0 (No magnetic force, No deviation) $\Rightarrow$ Neutral (q = 0)

(3) Negative charge = According to Right-Hand Rule negative charge will move in circular downward direction.

### Important

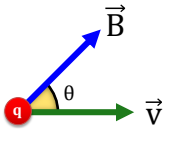
Work done by magnetic field on any charge is always **ZERO**!!

**Quick questions:** In presence of external Magnetic field(only)-

| | | |
|---|---|---|
| 1. | Can direction of a moving charge change | Yes |
| 2. | Can velocity of a moving charge change | Yes |
| 3. | Can a stationary charge start moving | No |

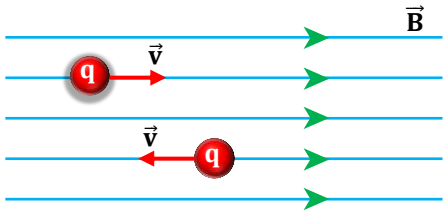
### Motion of Charged Particle in Uniform Magnetic Field

Based on angle between $\vec{v}$ and $\vec{B}$, charge can move on three type of paths in Uniform Magnetic Field




| Case 1 | Case 2 | Case 3 |
|---|---|---|
| $\theta = 0°/180°$ | $\theta = 90°$ | $\theta \neq 0°, 90°, 180°$ |
| Straight Line Path | Circular Path | Helical Path |

**Case 1:**




| θ = 0° | θ = 180° |
|---|---|
| F = qvB sin0° = 0 | F = qvB sin180° = 0 |
| Straight Line Path | Straight Line Path |

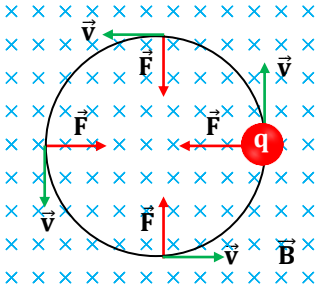
**Case 2: Motion of charge particle in uniform transverse magnetic field**

When a charge +q projected in uniform transverse magnetic field $\left(\theta = 90°, \vec{v} \perp \vec{B}\right)$ then maximum magnetic force of constant magnitude always acts perpendicular to its direction of motion so the charge moves along circular path and required centripetal force provided by the magnetic force.

Magnetic force on moving charge given as

$$\vec{F}_m = q\left(\vec{v} \times \vec{B}\right)$$

$$F_m = qvB\sin 90° = qvB$$




Since centripetal force is provided by magnetic force

$$F_m = F_{cp}$$

$$qvB = \frac{mv^2}{r}$$

$$R = \frac{mv}{qB} \qquad \text{(R : Radius of curvature)}$$

Now, momentum, p = mv

$$R = \frac{p}{qB} = \frac{\sqrt{2mK}}{qB} \qquad \text{(K : Kinetic energy)}$$

If a charge particle comes in motion from rest, due to an accelerating potential difference $V_{acc}$

Then, its KE, $K = qV_{acc}$

$$\text{So, } R = \frac{\sqrt{2mqV_{acc}}}{qB} \quad \Rightarrow \quad R = \frac{1}{B}\sqrt{\frac{2mV_{acc}}{q}}$$

**Time period (T) :–**

Time period of charge particle of a circular motion in uniform transverse magnetic field given as

$$T = \frac{2\pi R}{v} \text{ where } R = \frac{mv}{qB}$$

$$\text{So, } T = \frac{2\pi}{v}\left(\frac{mv}{qB}\right) = \frac{2\pi m}{qB}$$

$$\text{Frequency, } f = \frac{1}{T} = \frac{qB}{2\pi m}$$

$$\text{Angular frequency, } \omega = \frac{qB}{m}$$

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## Note:

- Time period does not depend on velocity or Radius!!
- It depends on ratio of charge to mass of the particle

| Particle | Charge | Mass | q/m |
|---|---|---|---|
| electron | e | $m_e = 9.1 \times 10^{-31}$ kg | $0.17 \times 10^{12}$ C/kg |
| proton | e | $m_p = 1.67 \times 10^{-27}$ kg | $0.95 \times 10^{8}$ C/kg |
| neutron | 0 | $m_p \approx m_p$ | 0 |
| deuteron | e | $2m_p$ | $0.47 \times 10^{8}$ C/kg |
| $\alpha$ -particle | 2e | $4m_p$ | $0.47 \times 10^{8}$ C/kg |

Charge to mass ratio is known as **specific charge.**

## Illustration 46:

Proton, deuteron and an $\alpha$ particle are moving $\perp$ to B then find the ratio of their radii if they are moving :-

1. with Same speed
2. with Same momentum
3. with Same Kinetic Energy
4. through Same accelerating potential

## Solution:

| Particle | Mass | Charge |
|---|---|---|
| Proton | $m_p$ | +e |
| Deutron | $2m_p$ | +e |
| Alpha | $4m_p$ | +2e |

**1.** $R = \frac{mv}{qB}$

$R \propto \frac{m}{q}$

$R_1 : R_2 : R_3 = \frac{m_1}{q_1} : \frac{m_2}{q_2} : \frac{m_3}{q_3} = 1 : 2 : 2$

**2.** $R = \frac{p}{qB}$

$R \propto \frac{1}{q}$ $(p \rightarrow \text{same})$

$R_1 : R_2 : R_3 = \frac{1}{q_1} : \frac{1}{q_2} : \frac{1}{q_3} = 1 : 1 : \frac{1}{2} = 2 : 2 : 1$

**3.** $R = \frac{\sqrt{2mK}}{qB}$ $(K \rightarrow \text{same})$

$R \propto \frac{\sqrt{m}}{q}$

$R_1 : R_2 : R_3 = \frac{\sqrt{m_1}}{q_1} : \frac{\sqrt{m_2}}{q_2} : \frac{\sqrt{m_3}}{q_3} = 1 : \sqrt{2} : 1$

**4.** $R = \frac{1}{B}\sqrt{\frac{2mV_{acc}}{q}}$

$R \propto \sqrt{\frac{m}{q}}$ $V_{acc.} \rightarrow \text{same}$

$R_1 : R_2 : R_3 = \sqrt{\frac{m_1}{q_1}} : \sqrt{\frac{m_2}{q_2}} : \sqrt{\frac{m_3}{q_3}} = 1 : \sqrt{2} : \sqrt{2} = 1 : \sqrt{2} : \sqrt{2}$

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**Illustration 47:**

A Deuteron of kinetic energy 100 keV is describing a circular orbit of radius 0.5 metre in a plane perpendicular to magnetic field $\vec{B}$. The kinetic energy of the proton that describes a circular orbit of radius 0.5 metre in the same plane with the same $\vec{B}$ is

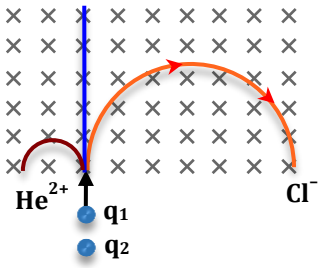
**Solution:**

$$r=\frac{\sqrt{2mK}}{qB}\Rightarrow K\propto\frac{q^2}{m}\Rightarrow\frac{K_p}{K_d}=\left(\frac{q_p}{q_d}\right)^2\times\frac{m_d}{m_p}=\left(\frac{1}{1}\right)^2\times\frac{2}{1}=\frac{2}{1}\Rightarrow k_p=2\times100=200\text{ keV}.$$

**Illustration 48:**

$He^{2+}$ and $Cl^-$, both are projected with same momentum in a transverse magnetic field. Which one will have more curvature?

**Solution:**




$$R\propto\frac{m}{q}$$

$$\text{Curvature (C)}\propto\frac{1}{\text{Radius of Curvature (R)}}$$

$$\left(\frac{m}{q}\right)_{Cl^-}>\left(\frac{m}{q}\right)_{He^{2+}}$$

So, $He^{2+}$ has more curvature.

**Illustration 49:**

Proton, deuteron and an α particle are moving perpendicular to B then find the ratio of their time periods.

**Solution:**

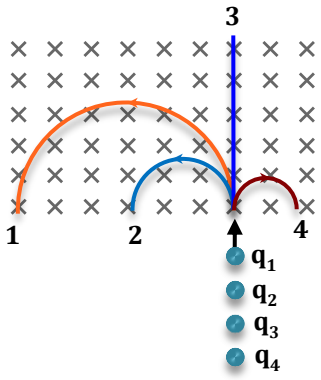
$$\text{Since, } T=\frac{2\pi m}{qB}, T\propto\frac{m}{q}$$

$$T_P:T_D:T_\alpha=\frac{m_P}{q_P}:\frac{m_D}{q_D}:\frac{m_\alpha}{q_\alpha}$$

$$=\frac{m}{q}:\frac{2m}{q}:\frac{4m}{2q}=1:2:2$$

**Illustration 50:**

Four charge particles proton, electron, neutron and a-particle projected with same velocity in same uniform transverse magnetic field identify according to the given diagram.




**Solution:**

$$R=\frac{mv}{qB},\ R\propto\frac{m}{q}$$

$$R_1:R_2:R_3:R_4::\frac{m_1}{q_1}:\frac{m_2}{q_2}:\frac{m_3}{q_3}:\frac{m_4}{q_4}$$

$$\text{Curvature (C)}\propto\frac{1}{\text{Radius of Curvature (R)}}$$

We identify in following graph with help of Relation (1) and (2)

1 → α – particle, 2 → proton, 3 → neutron, 4 → electron

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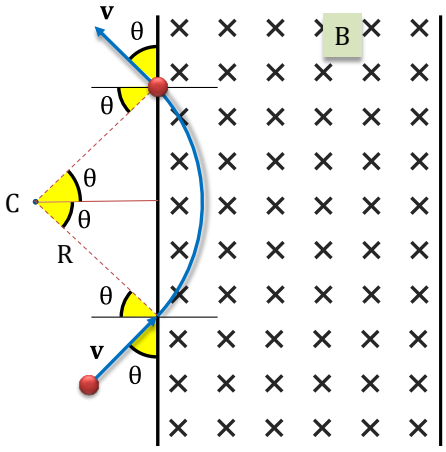
## Motion of Charge Particle in Limited Magnetic Field

A charge (q,m) is projected from **field free region to field region** of width 'd'

**Case-1:** For general angle of entrance $\theta$




1. Deviation angle $2\theta$
2. Time spent in M.F.

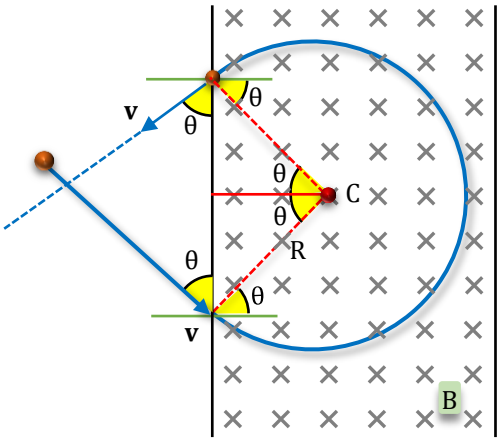
$$T' = T \times \frac{2\theta}{2\pi} \qquad \text{where } T = \frac{2\pi m}{qB}$$

**Or** $T' = T \times \frac{2\theta}{360^\circ}$

3. Displacement of particle

$\Delta s = 2R\sin\theta$

**Case-2:** For general angle of entrance $\theta$




1. Deviation angle $2\theta$
2. Time spent in M.F.

$$T' = T \times \frac{2\pi - 2\theta}{2\pi} \qquad \text{where } T = \frac{2\pi m}{qB}$$

**Or** $T' = T \times \frac{2\pi - 2\theta}{360^\circ}$

3. Displacement of particle

$\Delta s = 2r\sin\theta$

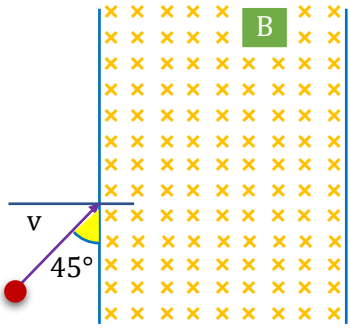
### Illustration 51:

A charge particle (q, m) enters in transverse B as shown in fig. Then find-

(1) Deviation angle

(2) Time for which particle remains in M.F.

(3) Displacement of particle when particle emerges out.




**Solution:**

(1) Deviation angle = 45°+ 45° = 90°

(2) Time for which particle remains in M.F. : -

We know that $\theta = \omega t$

$$\frac{\pi}{2} = \frac{2\pi}{T} \times t$$

$$\frac{\pi}{2} = \frac{2\pi}{\frac{2\pi m}{qB}} \times t \Rightarrow t = \frac{\pi m}{2qB}$$

(3) Displacement of particle when particle emerges out : -

$$= R\cos 45^\circ + R\cos 45^\circ = \sqrt{2}R = \sqrt{2}\frac{mv}{qB}$$

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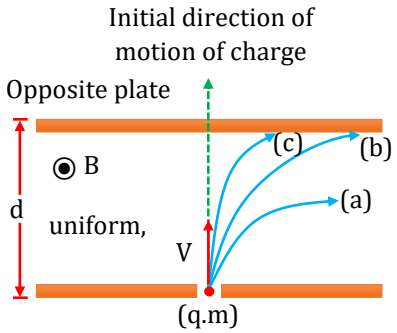
A charge (q,m) is projected from **field free region to field region** in such a way that its velocity (v) is perpendicular to the magnetic field (B). The charge deflected from its initial direction of motion and trace a part of circle according to following conditions: -



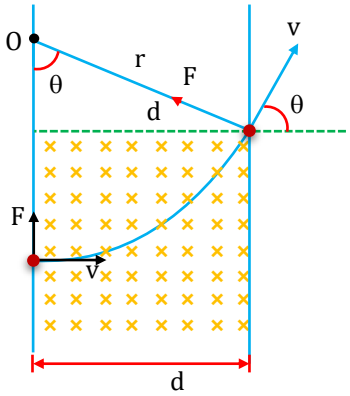

(a) Charge does not strike to the opposite plate and completes the semi-circle $\Rightarrow$ r < d

(b) Charge does not strike to the opposite plate and just completes the semi-circle $\Rightarrow$ r = d

(c) Charge strikes to the opposite plate and does not completes the semi-circle $\Rightarrow$ r > d

**Illustration 52:**

A particle of mass m and charge q is projected into a region having a perpendicular uniform magnetic field B of width $d < \frac{mv}{qB}$. Find the angle of deviation θ of the particle as it comes out of the magnetic field.




**Solution:**

The radius of the circular orbit is r = $\frac{mv}{qB}$

The deviation θ may be obtained from the fig. as

$\sin\theta = \frac{d}{r} = \frac{dBq}{mv}$ or $\theta = \sin^{-1}\left(\frac{dBq}{mv}\right)$

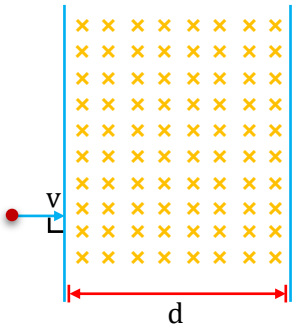
**Illustration 53:**

A charge particle (q, m) enters in transverse B as shown in fig. Then find-

(1) Deviation angle

(2) Time for which particle remains in M.F.

(3) Displacement of particle when particle emerges out. Given : $d > \frac{mv}{qB}$




**Solution:**

Here as d > r so particle completes the semi-circle

(1) 2θ = 180° = π

(2) $t = \frac{T}{2}$

(3) 2r

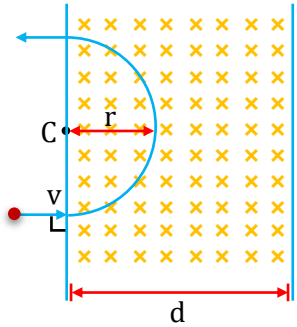
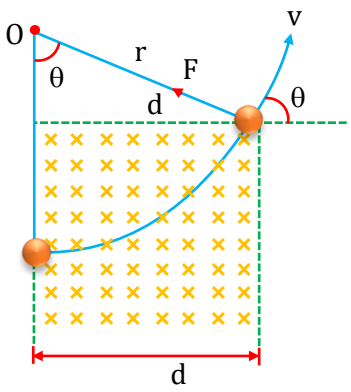


**Illustration 54:**

A proton accelerated by a potential difference 500 kV moves though a transverse magnetic field of 0.51T as shown in figure. The angle θ through which the proton deviates from the initial direction of its motion, is




**Solution:**

According to following figure, $\sin\theta = \frac{d}{r}$

Also, $r = \frac{\sqrt{2mK}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$

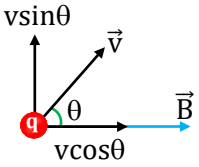
$\therefore \sin\theta = Bd\sqrt{\frac{q}{2mV}}$

$= 0.51\times 0.1\sqrt{\frac{1.6\times10^{-19}}{2\times1.67\times10^{-27}\times500\times10^{3}}} = \frac{1}{2} \Rightarrow \theta = 30°$

**Case 3: Motion of Charge Particle in Oblique Magnetic Field ($\theta \neq 0°, 90°, 180°$)**

Consider a charge 'q' of mass 'm' moving with velocity $\vec{v}$ at an angle θ with the direction of magnetic field $\vec{B}$.

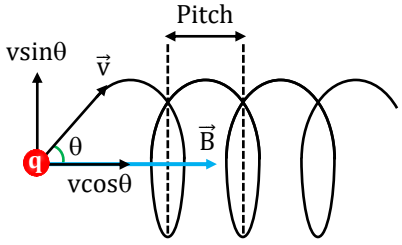
In this situation resolving the velocity of charge along and perpendicular to the field, we find that the particle moves with constant velocity vcosθ along the field (along x-axis) and at the same time it is also moving with velocity vsinθ perpendicular to the field due to which it will describe a circle in a plane perpendicular to the field (y–z plane).

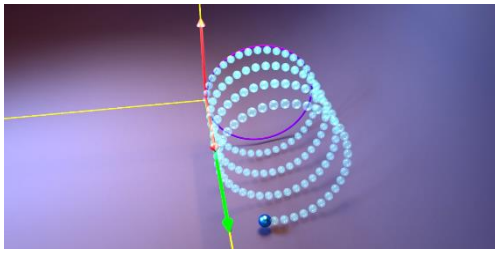



$v\sin\theta \perp \vec{B} \Rightarrow$ Circular motion

$v\cos\theta \| \vec{B} \Rightarrow$ Straight Line motion

So in the combined effect of both the component of velocity, charge will move along 'helical path' and the curve is called 'helix'.



**Axis of helix**

It is a straight line along external magnetic field which joins all the centres of circular turns, called axis of helix.

**Radius of Circular Path**

Centripetal force required to move the charge in a circle is provided by the magnetic force $F_m = F_{cp}$

$q(v\sin\theta)B\sin90° = \frac{m(v\sin\theta)^2}{R} \Rightarrow \boxed{R = \frac{mv\sin\theta}{qB}}$

**or**

$R = \frac{mv_{\perp}}{qB} \Rightarrow R = \frac{mv\sin\theta}{qB}$

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**Time period of circular motion**

$T = \frac{2\pi R}{v\sin\theta}$, where $R = \frac{mv\sin\theta}{qB}$

$T = \frac{2\pi}{v\sin\theta}\left(\frac{mv\sin\theta}{qB}\right) \Rightarrow \boxed{T = \frac{2\pi m}{qB}}$ (Time does not depend on v)

**Pitch of helix (p)**

The linear distance travelled by the charge particle in one revolution or in one time period along external magnetic field direction is called 'pitch of helix'.

Displacement in T time

$p = (v\cos\theta)T$, where $T = \frac{2\pi m}{qB} \Rightarrow \boxed{p = \frac{2\pi mv\cos\theta}{qB}}$

**Illustration 55:**

A charged particle moves in a uniform magnetic field. The velocity of the particle at some instant makes an acute angle with the magnetic field. The path of the particle will be

**Solution:**

When particle enters at an angle other than 0°, 90° or 180°, path followed is helix.

**Illustration 56:**

A charge (q, m) projected with velocity $\vec{v} = 3\hat{i} + 4\hat{j}$ m/s in external magnetic field $\vec{B} = 6\hat{i}$ T. Find

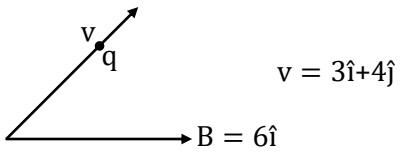
(i) Radius of circular path (ii) Pitch (iii) Time period

**Solution:**

$\sin\theta = \frac{4}{5}$

Radius of circular path.

$R = \frac{mv\sin\theta}{qB} \Rightarrow R = \frac{m\times 4\times\frac{4}{5}}{q\times 6}$




$v = 3\hat{i}+4\hat{j}$

Time period $= \frac{2\pi m}{6q}$

Pitch = $p = (v\cos\theta)T$

$p = 3\times\frac{2\pi m}{6q}$

**Illustration 57:**

A charged particle enters a uniform magnetic field with velocity vector at an angle of 45° with the magnetic field. The pitch of the helical path followed by the particle is p.

The radius of the helix will be

**Solution:**

$p = \frac{2\pi m}{Bq}(v\cos 45°) = \frac{2\pi m}{Bq}(v\sin 45°)$

$\therefore \frac{mv\sin 45°}{Bq} = \frac{p}{2\pi}$ = radius of helix

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### Illustration 58:

A proton of mass $1.67 \times 10^{-27}$ kg and charge $1.6 \times 10^{-19}$ C is projected with a speed of $2 \times 10^6$ m/s at an angle of 60° to the X-axis. if a uniform magnetic field of 0.104 T is applied along Y-axis, the path of proton is.

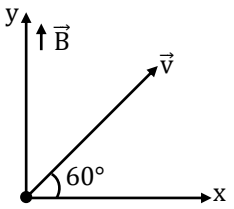
### Solution:

Path of the proton will be a helix of radius, $r = \frac{mv\sin\theta}{qB}$

(where $\theta$ = angle between $\vec{B}$ and $\vec{v}$ )

$\Rightarrow r = \frac{1.67\times10^{-27}\times2\times10^{6}\times\sin30°}{1.6\times10^{-19}\times0.104} = 0.1$ m

Time period, $T = \frac{2\pi m}{qB} = \frac{2\pi\times1.67\times10^{-27}}{1.6\times10^{-19}\times0.104} = 2\pi \times 10^{-7}$ s




### Lorentz Force

When a charge is moving in a region, where both electric field $\vec{E}$ and magnetic field $\vec{B}$ exist, then electric and magnetic forces are acting on it. The resultant of these forces called electromagnetic or Lorentz force on charge. (zero gravity)

**Net Electromagnetic force** is known as Lorentz force.

$$\vec{F}_{net} = \vec{F}_m + \vec{F}_e$$

$$\vec{F}_{net} = q(\vec{v}\times\vec{B}) + (q\vec{E})$$

- If $F_{net} = 0$
  Then v = constant

**Note :** If $\vec{E} \neq 0$ , $\vec{B} \neq 0$

and $\vec{F}_{net} = \vec{0}$

$$q(\vec{v}\times\vec{B}) + (q\vec{E}) = \vec{0}$$

$$\vec{E} = -(\vec{v}\times\vec{B})$$

for both positive and negative charges

### Illustration 59:

A beam of protons is deflected sideways. Could this deflection be caused by
(i) a magnetic field (ii) an electric field? If either possible, what would be the difference?

### Solution:

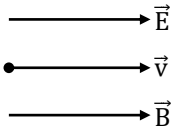
Yes, the moving charged particle (e.g. proton, a-particles etc.) may be deflected sideway either by an electric or by a magnetic field.

(i) The force exerted by a magnetic field on the moving charged particle is always perpendicular to direction of motion, so that no work is done on the particle by this magnetic force. That is the magnetic field simply deflects the particle and does not increases its kinetic energy.

(ii) The force exerted by electric field on the charged particle at rest or in motion is always along the direction of field and the kinetic energy of the particle changes.

### Illustration 60:

A particle moving with velocity of $\vec{v}$ as shown in figure then the path of trajectory and also find the effect on the speed.




### Solution:

Path will be straight line with increasing speed.

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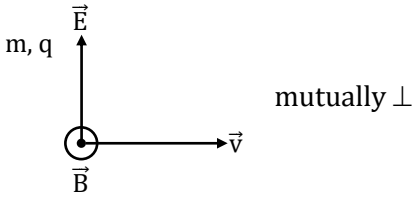
**Illustration 61:**

Find v such that particle moves constant velocity.




**Solution:**

$qE = qvB \Rightarrow v = \frac{E}{B}$

**Illustration 62:**

A particle of charge $-1.6 \times 10^{-18}$ coulomb moving with velocity 10 $ms^{-1}$ along the x-axis enters a region where a magnetic field of induction B is along the y-axis, and an electric field of magnitude $10^4$ V/m is along the negative z-axis. If the charged particle continues moving along the x-axis, the magnitude of B is

**Solution:**

Since particle is moving undeflected.

So $qE = qvB \Rightarrow B = E/v = \frac{10^4}{10} = 10^3 Wb/m^2$

**Illustration 63:**

An electron (mass = $9.1 \times 10^{-31}$ kg; charge = $1.6 \times 10^{-19}$ C) experiences no deflection if subjected to an electric field of $3.2 \times 10^5$ V/m, and a magnetic field of $2.0 \times 10^{-3}$ $Wb/m^2$. Both the fields are normal to the path of electron and to each other. If the electric field is removed, then the electron will revolve in an orbit of radius

**Solution:**

For no deflection in mutually perpendicular electric and magnetic field $v = \frac{E}{B} = \frac{3.2\times10^5}{2\times10^{-3}} = 1.6\times10^8 \text{ m/s}$.

If electric field is removed then due to only magnetic field radius of the path described by electron

$r = \frac{mv}{qB} = \frac{9.1\times10^{-31}\times1.6\times10^8}{1.6\times10^{-19}\times2\times10^{-3}} = 0.45\text{m}$

**Illustration 64:**

A particle having charge +5C moves with $\vec{v} = 3\hat{j}$ m/s in an electromagnetic field. If $\vec{B} = 3\hat{i} + 2\hat{j}$ T, and $\vec{E} = 2\hat{k}$, then find net electromagnetic force on the particle.

**Solution:**

Net Electromagnetic Force = Magnetic Force + Electric Force = $q(\vec{v}\times\vec{B}) + q\vec{E}$

$= 5(-9\hat{k}) + 5(2\hat{k}) = -35\hat{k}$

**Illustration 65:**

In a region uniform E.F & M.F are at along x-axis. A charge particle Q is projected towards the y direction with velocity v. Then find time in which K.E. of particle become twice.

**Solution:**

because $KE_f = 2KE_i$

$v' = \sqrt{2}v$

$F_e = qE = ma$

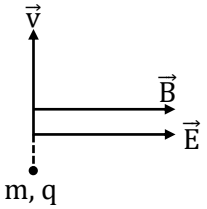
$a_x = \frac{qE}{m}$

Apply Law of motion in x direction

V' = V + at

$\sqrt{2}V = V + \frac{qE}{m}t$

$t = \frac{(\sqrt{2}-1)mV}{qE}$

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## Cyclotron

The cyclotron is a device which is used to accelerate the charge particles or ions to high energies. It was invented by E.O. Lawrence and M.S. Livingston in 1934.

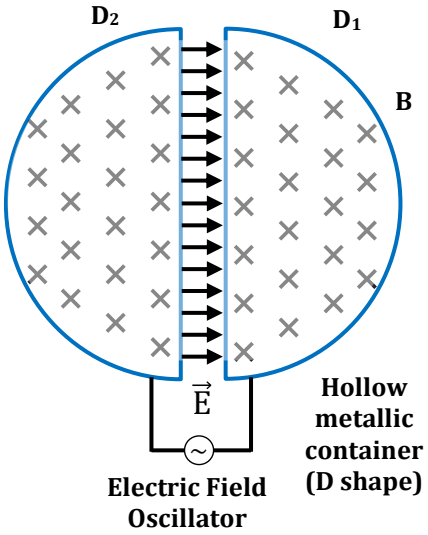
It is used to speed up POSITIVE CHARGED PARTICLES to acquire high kinetic energy for **nuclear reactions**.

### Principle

When a charged particle or ion is made to move again and again in a high frequency electric field and strong magnetic field, it gets accelerated and acquires sufficiently large amount of energy.

### Construction

It consists of two hollow D–shaped metallic chambers $D_1$ and $D_2$ called dees. These dees are separated by a small gap, where a source (P) of charged particles or ions is placed. Dees are connected to high frequency oscillator, which provides high frequency electric field across the gap of the dees. This arrangement is placed between the two poles of a strong electromagnet. The magnetic field due to this electromagnet is perpendicular to the plane of the dees.



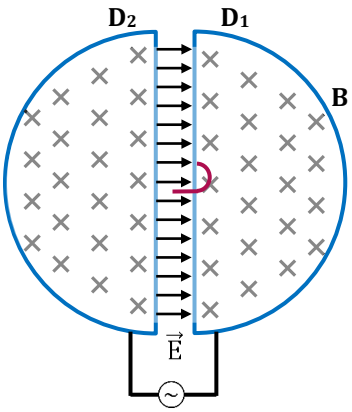

### Working

1. Charge is released at rest between two Dees. Due to electric field, +ve charge starts moving and achieves a speed and enters in $D_1$.
2. In transverse magnetic field, particle undergo circular motion.

   $R = \frac{mv}{qB}$ and $T = \frac{2\pi m}{qB}$

   Time spent in magnetic field during half revolution is T/2.

   Cyclotron frequency, $f_c = \frac{1}{T} = \frac{qB}{2\pi m}$

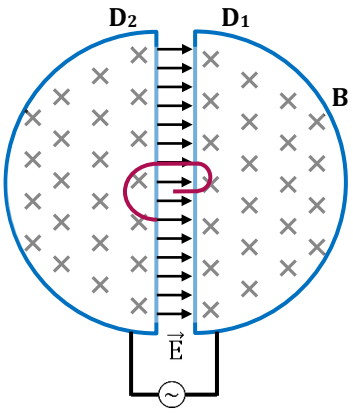



3. As charge again enters the space between the Dees, direction of electric field reverses so that $\vec{F} = q\vec{E} \| \vec{v}$ and speed of particle again increases and with increased speed, particle enters $D_2$.

   The sign of the electric field is changed alternately in tune with the circular motion of the particle. This ensures that the particle is always accelerated by the electric field.

   To ensure this cyclotron frequency fc must be equal to oscillator frequency (f) i.e. $f_c = f$.

   This is called resonance condition.




4. In transverse magnetic field, particle again undergoes circular motion with higher radius (due to increased speed).

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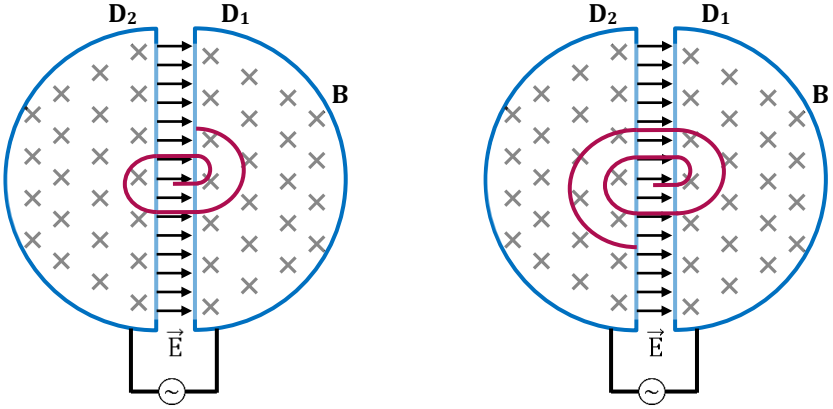
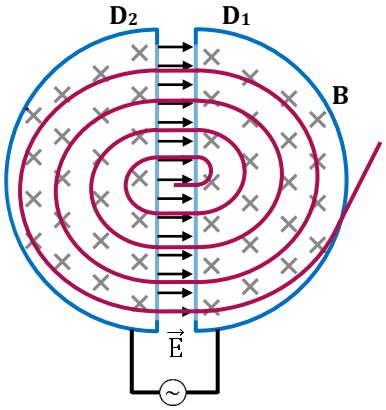
5. So, every time particle passes through the space between Dees, its speed increases and direction of motion is reversed by undergoing semi-circular motion in the Dees.




6. When the particle goes in circular motion with maximum radius, it is taken out of the cyclotron with $v_{max}$.




7. Maximum kinetic energy gained by the charge: -

$(E_K)_{max.} = \left(\frac{q^2B^2}{2m}\right) r_{max.}^2$ where $r_{max.}$ is radius of the longest semi-circle described by the ion.

8. Total energy acquired by the charge during 'N' complete revolution:

$E_T = 2NqV$, where V is potential difference to which the charge is subjected every time, it enters a dee. This energy must be equal to the maximum kinetic energy of charge:

$$2NqV = \frac{B^2q^2r_{max}^2}{2m} \Rightarrow N = \frac{B^2q\,r_{max}^2}{4Vm}$$

**Limitations of Cyclotron**

(i) Cyclotron is not used to accelerate uncharged particles like neutron.

(ii) Cyclotron is not used to accelerates electrons because they have very small mass.

It is used to speed up Positive Charged Particles to acquire high kinetic energy for nuclear reactions.

**Illustration 66:**

Cyclotron can not accelerate.

**Solution:**

Used to accelerate heavy positive charges like proton, deuteron, $\alpha$–particle.

Electron cannot be accelerated due to very small mass.

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**Illustration 67:**

In Cyclotron maximum radius is directly proportional to the

**Solution:**

$$R_{max} = \frac{mv_{max}}{qB} = \frac{P}{qB}$$

**Illustration 68:**

In a cyclotron, charge particle is always under acceleration. Explain

**Solution:**

In a cyclotron, charged particle experiences coulombic force (force due to electric field) between the Dees and magnetic force (force due to magnetic field) while circulating inside the Dees. In total, it always experiences a force so always accelerated.

**Illustration 69:**

In a cyclotron, if a deuteron can gain an energy of 60 MeV, then a $\alpha$ – particle can gain an energy of: -

**Solution:**

Energy in cyclotron $E = \frac{q^2B^2r^2}{2m}$

so $E \propto \frac{q^2}{2m} \Rightarrow \frac{E_1}{E_2} = \left(\frac{q_1}{q_2}\right)^2\left(\frac{m_2}{m_1}\right)$

For deuteron & $\alpha$ – particle $q_1 = e$, $q_2 = 2e$, $m_1 = 2$ amu, $m_2 = 4$ amu

so $\frac{E_1}{E_2} = \frac{1}{2} \Rightarrow E_2 = 2E_1 = 120$ MeV

## BEGINNER'S BOX-5

**1.** A test charge $1{\cdot}6 \times 10^{-19}$ Cb is moving with velocity $\vec{v} = (2\hat{i} + 3\hat{j})$ m/sec is a magnetic field $\vec{B} = (2\hat{i} + 3\hat{j})$ Wb/m$^2$. The magnetic force on the test charge :-

(1) $6\hat{k}$T (2) $\left(4\hat{i} + 6\hat{j}\right)$T (3) $\left(4\hat{i} + 6\hat{j}\right) \times 10^{-19}$T (4) zero

**2.** A charge with $10^{-11}$ Cb and $10^{-7}$ kg mass moving with a velocity of $10^8$ m/sec along x–axis. A uniform static magnetic field of 0·5T is acting along the y–axis. The magnetic force (magnitude and direction) on charge :-

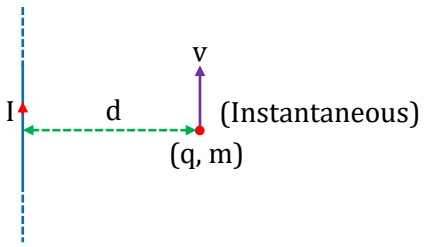
(1) zero (2) $5 \times 10^{-4}$N, along z-axis

(3) $5 \times 10^{-4}$ N, along x-axis (4) $5 \times 10^{-4}$ N, along y-axis

**3.** A 2MeV proton is moving perpendicular to uniform magnetic field of 2·5T. The magnetic force on the proton :-

(1) $8 \times 10^{-12}$ N (2) $4 \times 10^{-12}$ N (3) $16 \times 10^{-12}$ N (4) $2 \times 10^{-12}$ N

**4.** An electron is moving at $10^6$ m/sec in a direction parallel to a current of 5A flowing through an infinite long straight wire separated by a perpendicular distance of 10cm in air. Magnetic force experienced by the electron :-




(1) $1.6 \times 10^{-19}$ N

(2) $1.6 \times 10^{-20}$ N

(3) $1.6 \times 10^{-18}$ N

(4) $1.6 \times 10^{-21}$ N

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**5.** Three charge proton, deutron and α–particle are projected in uniform transverse magnetic field, then find ratio of their radii of circular tracks respectively, if their –
(a) v → same (b) P → same (c) $E_K$ → same (d) $V_{acc.}$ → same

**6.** Three charge proton, deutron and α–particle are projected in uniform transverse magnetic field in such a way that their radii of circular tracks are equal then find their following ratio respectively :-
(a) v → ratio (b) P → ratio (c) $E_K$ → ratio (d) $V_{acc.}$ → ratio

**7.** A proton and an electron are projected in uniform transverse magnetic field, then trajectory of which charge is more curved or deflection of which charge is more from its initial direction of motion, if their
(a) v → same (b) P → same (c) $E_K$ → same (d) $V_{acc.}$ → same

**8.** Three charge proton, deutron and α–particle are projected in uniform transverse magnetic field then find ratio of following parameters of circular motion respectively.
(a) Time period (b) Frequency (c) Angular frequency

**9.** A beam of proton moving with velocity $4 \times 10^5$ m/sec enters in a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. Calculate radius of helical path and pitch of helix.

**10.** A proton is projected with a speed of $2 \times 10^6$ m/sec at an angle of 60° to the x-axis. If a uniform magnetic field of 0.104T is applied along y axis. Calculate radius of helix and time period of proton.

**11.** A charge projected in uniform magnetic field at angle 30° to the field direction. The time period of circular motion of helical path is 'T'. If angle of projection becomes 60° then what is the new time period.

**12.** If a charge projected in a zero gravity region. Find possible cases of electric and magnetic field in that region so no net force exerts on charge.
(a) E = 0, B = 0 (b) E = 0, B ≠ 0 (c) E ≠ 0, B ≠ 0 (d) E ≠ 0, B = 0

**13.** If a projected charge passes through a zero gravity region without change in its velocity, then find possible cases of electric and magnetic field in that region
(a) E = 0, B = 0 (b) E = 0, B ≠ 0 (c) E ≠ 0, B ≠ 0 (d) E ≠ 0, B = 0

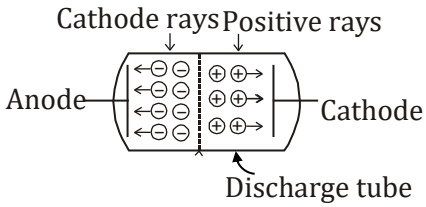
**14.** If a projected charge passes through a zero gravity region, remains undeviated or undeflected, then find possible cases of electric field and magnetic field in that region
(a) E = 0, B = 0 (b) E = 0, B ≠ 0 (c) E ≠ 0, B ≠ 0 (d) E ≠ 0, B = 0

**15.** If a projected charge passes through a zero gravity region, its speed may be constant, then find possible cases of electric and magnetic field in that region
(a) E = 0, B = 0 (b) E = 0, B ≠ 0 (c) E ≠ 0, B ≠ 0 (d) E ≠ 0, B = 0

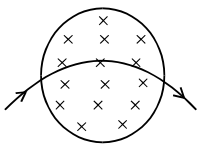
**16.** In a region a uniform magnetic field acts in horizontal plane towards north. If cosmic particles (80% protons) falling vertically downwards, then they are deflected towards–
(1) North (2) South (3) East (4) West

**17.** An electron is moving along +x direction. To get it moving along an anticlockwise circular path in x–y plane, magnetic field applied along :-
(1) +y–direction (2) +z–direction (3) –y–direction (4) –z–direction

**18.** A magnetic field is applied at perpendicular to the axis of cylindrical discharge tube then cathode and positive rays deflected in –
(1) Same direction.
(2) Opposite direction.
(3) Mutually perpendicular direction.
(4) No deflection for both.




**19.** There is a magnetic field acting in a plane perpendicular downwards. A particle in vacuum moves in the plane of paper from left to right as shown in figure. The path indicated by the arrow could be due to –
(1) Proton (2) Neutron
(3) Electron (4) Alpha particle



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**20.** A proton is moving towards north along horizontal line passes through a zero gravity region, where electric and magnetic field are mutually perpendicular to each other. The path of particle remains undeviated or undeflected. If the direction of electric field is towards east then direction of magnetic field is

(1) Towards east (2) Vertically downwards

(3) Towards west (4) Vertically upwards

**21.** A proton moving along z-axis with constant velocity. If a magnetic field is applied along x-axis then direction of magnetic force on proton

(1) along z-axis (2) along y-axis (3) along x-axis (4) zero force

**22.** A vertical wire carries a current in upward direction. If an electron beam sent horizontally towards the wire, then it will deflected

(1) vertically downwards and perpendicular to the plane of the paper

(2) vertically upwards and perpendicular to the plane of the paper

(3) In the plane of the paper

(4) No deflection

**23.** A charge is released from rest in a region of steady and uniform electric and magnetic fields which are parallel to each other. The charge will moves along which path :-

(1) Circular (2) Helical (3) Parabola (4) Straight line

## Magnetic Force on a Current Carrying Wire

### Current Carrying wire in Uniform Magnetic Field

When a current carrying conductor placed in magnetic field, a magnetic force exerts on each free electron which are present inside the conductor. The resultant of these forces on all the free electrons is called magnetic force on conductor.

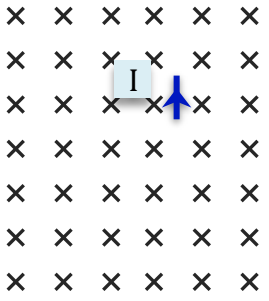
Force on a charge element dq moving in MF.

$$d\vec{F}_m = dq\left(\vec{v}\times\vec{B}\right)$$

$$d\vec{F}_m = dq\left(\frac{d\vec{\ell}}{dt}\times\vec{B}\right)$$

$$d\vec{F}_m = \frac{dq}{dt}\left(d\vec{\ell}\times\vec{B}\right)$$

$$d\vec{F}_m = I\left(d\vec{\ell}\times\vec{B}\right)$$




When current element $Id\vec{\ell}$ is placed in magnetic field $\vec{B}$, it experiences a magnetic force

$$d\vec{F}_m = I(d\vec{\ell}\times\vec{B})$$

- Net force on wire due to all such current elements

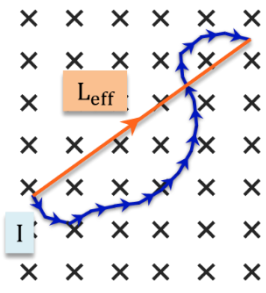
  $$\vec{F}_m = I\int\left(d\vec{\ell}\times\vec{B}\right)$$

- If B is uniform,

  $$\vec{F}_m = I\left(\int d\vec{\ell}\right)\times\vec{B}$$

  $$\Rightarrow \quad \vec{F}_m = I\left(\vec{L}_{eff}\times\vec{B}\right)$$

  $L_{eff}$ is the displacement vector from starting point of current to end point of current.

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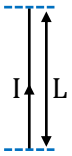
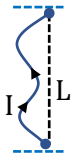
**Example**

For straight wire, $|L_{eff}| = L$




Arbitrary shape current carrying conductor $|L_{eff}| = L$




For loop, $|L_{eff}| = 0$

- Current element in a magnetic field does not experience any force if the current in it is parallel or anti–parallel with the field $\theta = 0°$ or $180°$ $dF_m = 0$ (min.)
- Current element in a magnetic field experiences maximum force if the current in it is perpendicular with the field $\theta = 90°$ $dF_m = BId\ell$ (max.)
- Magnetic force on current element is always perpendicular to the current element vector and magnetic field vector. $d\vec{F}_m \perp Id\vec{\ell}$ and $d\vec{F}_m \perp \vec{B}$ (always)

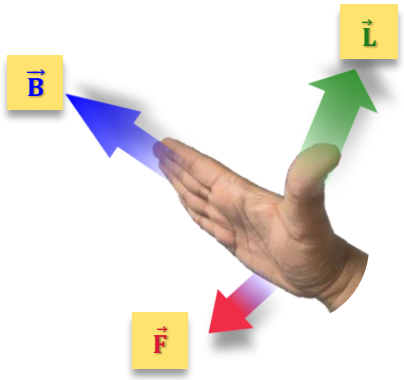
**Direction of magnetic force**

Magnetic force on the wire we can find by Right Hand Palm Rule

Fingers $\rightarrow$ In direction of external Magnetic field

Thumb $\rightarrow$ In direction of $\vec{L}$

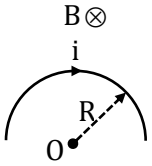
$\perp$ to Palm $\rightarrow$ Direction of magnetic force on wire




**Note :** For straight wire, direction of $\vec{L}$ is in the direction of current flow.

**Illustration 70:**

Find force on given wire in uniform magnetic field




**Solution:**

$\Rightarrow \quad \vec{F}_m = I\left(\vec{L}_{eff} \times \vec{B}\right) = I(L_{eff})B$

$= I(2R)B$ (Upward)

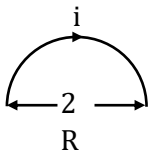



**Illustration 71:**

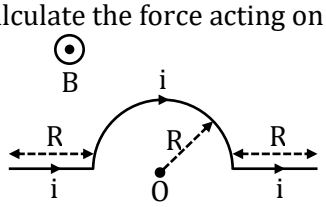
A current of 10 amperes is flowing in a wire of length 1.5 m. A force of 15 N acts on it when it is placed in a uniform magnetic field of 2 tesla. The angle between the magnetic field and the direction of the current is

**Solution:**

$$F = BiL\sin\theta \Rightarrow \sin\theta = \frac{F}{BiL} = \frac{15}{2\times10\times1.5} = \frac{1}{2} \Rightarrow \theta = 30°$$

**Illustration 72:**

A wire bent as shown in fig carries a current i and is placed in a uniform field of magnetic induction $\vec{B}$ that emerges from the plane of the figure. Calculate the force acting on the wire.




**Solution:**

The total force on the whole wire is $F_m = I|\vec{L}|B = I(R + 2R + R)B = 4RIB$

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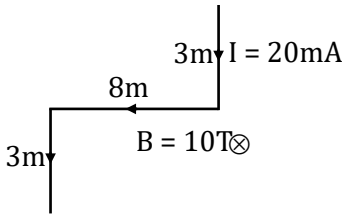
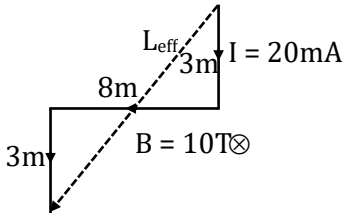
**Illustration 73:**
Find force on given wire.




**Solution:**




$L_{eff}$ = 10 m

$F_m = IL_{eff}B = 20 \times 10^{-3} \times 10 \times 10 = 2N$

## Current Carrying Loop in Uniform Magnetic Field

**Force :** Net force on a closed loop(any shape) in uniform magnetic field is always ZERO.

**Tension Developed in the wire :**

**T = RBI**

If length of wire is given

$$\ell = 2\pi R \quad \Rightarrow \quad R = \frac{\ell}{2\pi}$$

**Note:** Due to Tension in wire, loop of any shape becomes circular in shape.

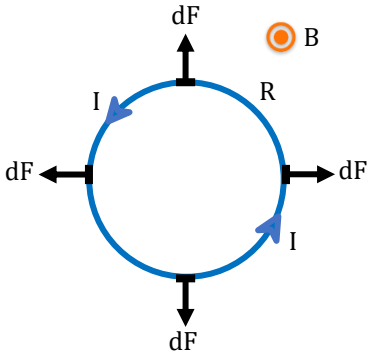



**Illustration 74:**

A conducting circular loop of radius r carries a constant current i. It is placed in a uniform magnetic field $\vec{B}$, such that $\vec{B}$ is perpendicular to the plane of the loop. The magnetic force acting on the loop is

**Solution:**

Net force on a current carrying closed loop is always zero, if it is placed in an uniform magnetic field.

**Illustration 75:**

A loop of flexible conducting wire of length 6.28 cm lies in magnetic field 2T is normal to plane of loop. A current 2A is passed through loop. The tension developed in wire to open up is

**Solution:**

T = RBI

$$\therefore \{\ell = 2\pi R\} = \frac{\ell}{2\pi}BI = \frac{6.28\times10^{-2}\times2\times2}{2\pi} = 4\times10^{-2}N$$

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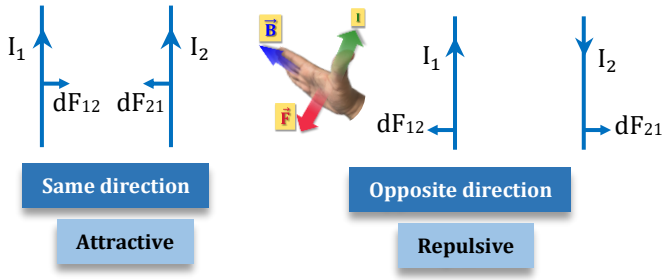
## Magnetic Force Between Two Parallel Current Carrying Wires

### Nature of Force

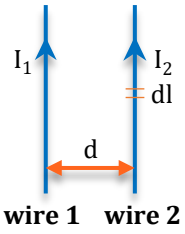



### Magnitude of Force

The net magnetic force acts on a current carrying conductor due to its own field is zero. So consider two infinite long parallel conductors separated by distance 'd' carrying currents $I_1$ and $I_2$.

Magnetic field at each point on conductor (ii) due to current $I_1$ is

$$B_1 = \frac{\mu_0 I_1}{2\pi d} \text{ [uniform field for conductor (2)]}$$




Magnetic field at each point on conductor (i) due to current $I_2$ is

$$B_2 = \frac{\mu_0 I_2}{2\pi d} \text{ [Uniform field for conductor (1)]}$$

Consider a small element of length ' $d\ell$ ' on each conductor. These elements are right angle to the external magnetic field, so magnetic force experienced by elements of each conductor given as

$$dF_{12} = B_2 I_1 \, d\ell = \left(\frac{\mu_0 I_2}{2\pi d}\right) I_1 \, d\ell \qquad \text{...(i)} \qquad (\text{Where } I_1 \, d\ell \perp B_2)$$

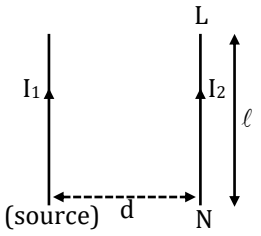
$$dF_{21} = B_1 I_2 \, d\ell = \left(\frac{\mu_0 I_1}{2\pi d}\right) I_2 \, d\ell \qquad \text{...(ii)} \qquad (\text{Where } I_2 \, d\ell \perp B_1)$$

Where $dF_{12}$ is magnetic force on element of conductor (i), due field of conductor (ii) and $dF_{21}$ is magnetic force on element of conductor (ii), due to field of conductor (i).

**Magnetic force per unit length of each conductor is** $\frac{dF_{12}}{d\ell} = \frac{dF_{21}}{d\ell} = \frac{\mu_0 I_1 I_2}{2\pi d}$

$f = \frac{\mu_0 I_1 I_2}{2\pi d}$ N/m (in S.I.) $\quad f = \frac{2 I_1 I_2}{d}$ dyne/cm (In C.G.S.)

Force scale $f = \frac{\mu_0 I_1 I_2}{2\pi d}$ is applicable when at least one conductor must be of infinite length so it behaves like source of uniform magnetic field for other conductor.




Magnetic force on conductor 'LN' is $F_{LN} = f \times \ell \Rightarrow F_{LN} = \left(\frac{\mu_0 I_1 I_2}{2\pi d}\right)\ell$

### Definition of Ampere

Magnetic force/unit length for both infinite length conductor gives as

$$f = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{(4\pi \times 10^{-7})(1)(1)}{2\pi(1)} = 2 \times 10^{-7} \text{ N/m}$$

'Ampere' is the current which, when passed through each of two parallel infinite long straight conductors placed in free space at a distance of 1 m from each other, produces between them a force of $2 \times 10^{-7}$ N/m

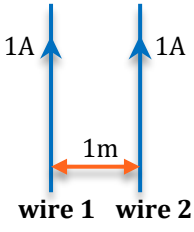



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**Illustration 76:**

Through two parallel wires A and B, 10A and 2A of currents are passed respectively in opposite direction. If the wire A is infinitely long and the length of the wire B is 2 m, the force on the conductor B, which is situated at 10 cm distance from A will be

**Solution:**

$$F = \frac{\mu_0}{4\pi}.\frac{2i_1i_2}{a}\times l \Rightarrow F = 10^{-7}\times\frac{2\times10\times2}{(10\times10^{-2})}\times2 = 8\times10^{-5}\,N$$

**Illustration 77:**

Two long and parallel wires are at a distance of 0.1 m and a current of 5 A is flowing in each of these wires. The force per unit length due to these wires will be

**Solution:**

$$F = \frac{\mu_0}{4\pi}\frac{2\times i_1i_2}{a} = \frac{10^{-7}\times2\times5\times5}{0.1} = 5\times10^{-5}\,N/m$$

**Illustration 78:**

A current of 5A flows through two long parallel wires. The magnetic force on each wire is $4 \times 10^{-3}$ N/m. If their currents makes half and separation between them makes double then magnetic force per unit length of each wire becomes :-

**Solution:**

Magnetic force per unit length

$$f_m = \frac{\mu_0 I_1 I_2}{2\pi d} \qquad ...(1)$$

$$f_m' = \frac{\mu_0\left(\frac{I_1}{2}\right)\left(\frac{I_2}{2}\right)}{2\pi(2d)} = \frac{\mu_0 I_1 I_2}{2\pi d}\times\frac{1}{8} \qquad ...(2)$$

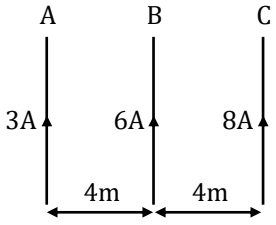
From (1) & (2)

$$f_m' = \frac{f_m}{8} = \frac{4\times10^{-3}}{8} = 0.5\times10^{-3}\,N/m$$

**Illustration 79:**

Three long straight wires, carrying currents are arranged according to figure.

Magnetic force on 20cm part of the wire B is :-




**Solution:**

Resultant force on '$\ell$' length of wire B

$F_{net} = (F_C - F_A)\,\ell$, $(F_C > F_A)$

$$= \left[\frac{\mu_0(8)(6)}{2\pi(4)} - \frac{\mu_0(3)(6)}{2\pi(2)}\right]\times 20\times10^{-2}$$

$= 1.2 \times 10^{-7}$ N, towards right

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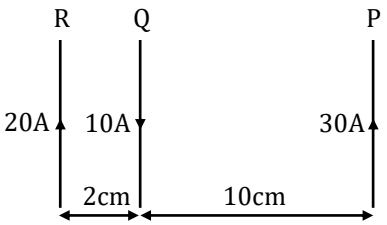
**Illustration 80:**

Three long, straight and parallel wires carrying currents are arranged as shown in figure. The force experienced by 10 cm length of wire Q is




**Solution:**

Force on wire Q due to wire P is

$$F_P = 10^{-7} \times \frac{2\times 30\times 10}{0.1} \times 0.1 = 6\times 10^{-5}\,N \quad \text{(Towards left)}$$

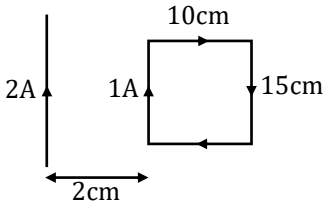
Force on wire Q due to wire R is

$$F_R = 10^{-7} \times \frac{2\times 20\times 10}{0.02} \times 0.1 = 20\times 10^{-5}\,N \quad \text{(Towards right)}$$

Hence $F_{net} = F_R - F_P = 14\times 10^{-5}\,N = 1.4\times 10^{-4}\,N$ (Towards right)

**Illustration 81:**

What is the net force on the square coil

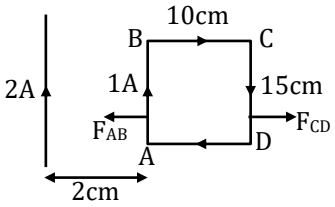



**Solution:**

Force on side BC and AD are equal but opposite so their net will be zero.

But $F_{AB} = 10^{-7} \times \frac{2\times 2\times 1}{2\times 10^{-2}} \times 15\times 10^{-2} = 3\times 10^{-6}\,N$

and $F_{CD} = 10^{-7} \times \frac{2\times 2\times 1}{\left(12\times 10^{-2}\right)} \times 15\times 10^{-2} = 0.5\times 10^{-6}\,N$

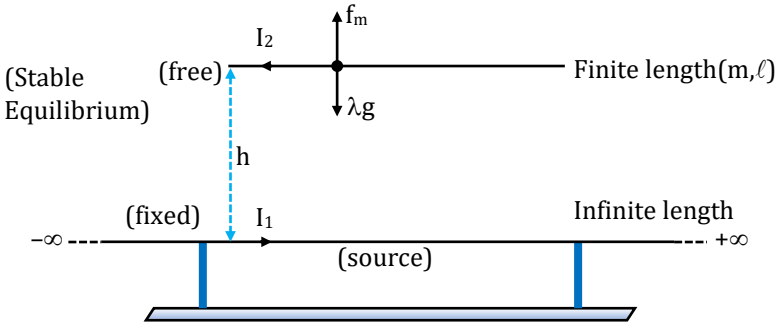



$\Rightarrow F_{net} = F_{AB} - F_{CD} = 2.5\times 10^{-6}\,N$

$= 25\times 10^{-7}\,N$ , towards the wire.

**Equilibrium of free wire**

**Case I :** Upper wire is free : Consider a long horizontal wire which is rigidly fixed another wire is placed directly above and parallel to fixed wire.



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Magnetic force per unit length of free wire $f_m = \frac{\mu_0 I_1 I_2}{2\pi h}$, and it is repulsive in nature because currents are unlike.

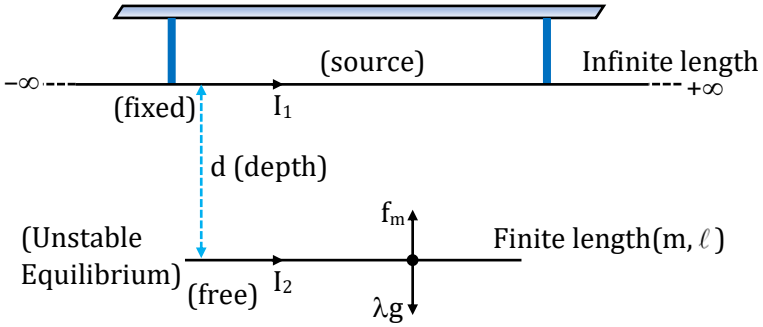
Free wire may remains suspended if the magnetic force per unit length is equal to weight of its unit length

At balanced condition $f_m = W'$. Weight per unit length of free wire = $\frac{\mu_0 I_1 I_2}{2\pi h} = \frac{m}{\ell} g$ (stable equilibrium condition)

If free wire is slightly displaced and released then it will executes S.H.M. in vertical plane.

The time period of motion is $T = 2\pi\sqrt{\frac{h}{g}}$

**Case II :** Lower wire is free : Consider a long horizontal wire which is rigidly fixed. Another wire is placed directly below and parallel to the fixed wire.




Magnetic force per unit length of free wire is $f_m = \frac{\mu_0 I_1 I_2}{2\pi d}$, and it is attractive in nature because currents are like.

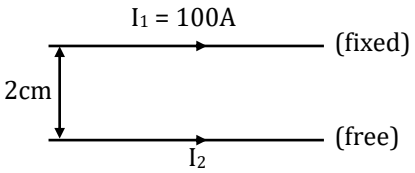
Free wire may remains suspended if the magnetic force per unit length is equal to weight of its unit length

At balanced condition $f_m = W'$

Weight per unit length of free wire $\frac{\mu_0 I_1 I_2}{2\pi d} = \frac{m}{\ell} g$ (unstable equilibrium condition)

### Illustration 82:

A long horizontal wire is rigidly fixed and carries 100A current. Another wire of linear mass density $2 \times 10^{-3}$ kg/m placed below and parallel to the fixed wire. If the free wire kept 2cm below and hangs in air, then current in free wire is:-




### Solution:

At balanced condition of free wire :-

$f_m = \lambda g$

$\frac{\mu_0 I_1 I_2}{2\pi d} = \lambda g$

$I_2 = \frac{2\pi\lambda g d}{\mu_0 I_1}$ $(\lambda = 2 \times 10^{-3}$ kg/m$) = \frac{2 \times 3.14 \times 2 \times 10^{-3} \times 9.8 \times 2 \times 10^{-2}}{4\pi \times 10^{-7} \times 100} = 19.6$ A (in free wire)

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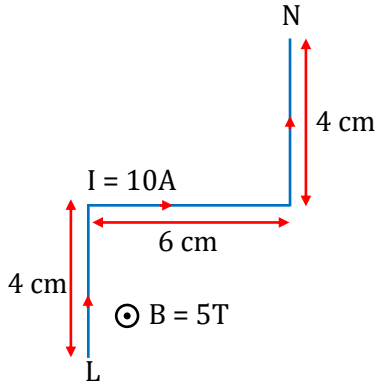
## BEGINNER'S BOX-6

**1.** A wire of length 5 cm. is placed inside the solenoid near its centre such that it makes an angle of 30° with the axis of solenoid. The wire carries a current of 5A and the magnetic field due to solenoid is $2.5 \times 10^{-2}$ T. Magnetic force on the wire is :-

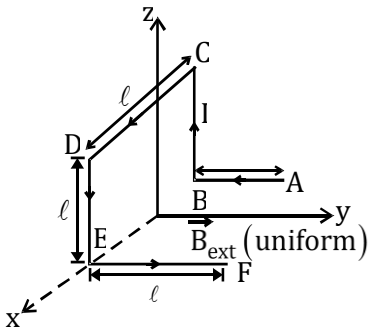
(1) $3.12 \times 10^{-4}$ N (2) $31.2 \times 10^{-4}$ N (3) $312 \times 10^{-4}$ N (4) $0.312 \times 10^{-4}$ N

**2.** A wire 'LN' bent as shown in figure is placed in uniform perpendicular magnetic field of 5T. A 10A current flows through the wire. Magnetic force experienced by the wire is :-




(1) 5N (2) 10N (3) 2.5 N (4) 1.25 N

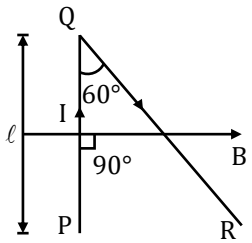
**3.** A wire ABCDEF with each side of length '$\ell$' bent as shown in figure and carrying a current I. If it is placed in a uniform magnetic field B which is parallel to +y direction. Magnetic force experienced by the wire is:-




(1) BI$\ell$, along + z direction (2) BI$\ell$, along –z direction

(3) 2BI$\ell$, along + z direction (4) 2BI$\ell$, along –z direction

**4.** A current 'I' is flowing through wire PQR. This wire is bent in form of an angle and placed is uniform magnetic field 'B' according to figure. If PQ = $\ell$ and $\angle$PQR = 60°. The ratio of magnetic forces on PQ to QR respectively:-




(1) 1 : 2 (2) $\sqrt{3}$:2 (3) 2 : $\sqrt{3}$ (4) 1 : 1

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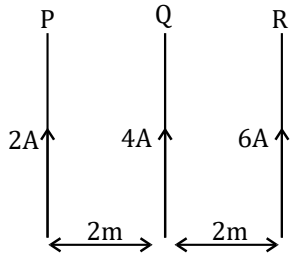
**5.** A current of 10A flows through two long parallel wires. The magnetic force on each wire is $2 \times 10^{-3}$N/m. If their currents makes doubled and separation between them makes half then magnetic force per unit length of each wire becomes :-

(1) $16 \times 10^{-3}$ N/m (2) $8 \times 10^{-3}$ N/m (3) $4 \times 10^{-3}$ N/m (4) $32 \times 10^{-3}$ N/m

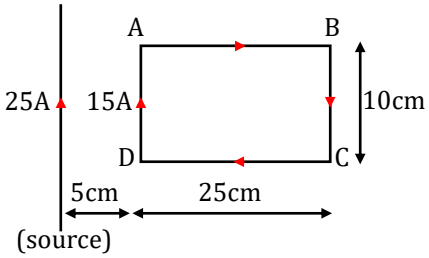
**6.** Three long straight wires, carrying currents are arranged according to figure. Magnetic force on 10cm part of the wire Q is :-




(1) $16 \times 10^{-9}$ N, towards right (2) $16 \times 10^{-8}$ N, towards right

(3) $16 \times 10^{-8}$ N, towards left (4) $16 \times 10^{-9}$ N, towards left

**7.** A rectangular loop ABCD is placed near on infinite length current carrying wire. Magnetic force on the loop is :-




(1) $1.25 \times 10^{-4}$ N, Attraction (2) $1.25 \times 10^{-4}$ N, Repulsion

(3) $12.5 \times 10^{-4}$ N, Repulsion (4) $12.5 \times 10^{-4}$ N, Attraction

**8.** A long horizontal wire 'P' which is rigidly fixed, carries a current 50A. An another wire 'Q' is placed directly above and parallel to 'P'. The weight of wire Q is 0.075 N/m and carries a current of 25A. The position of wire 'Q' from P so that the wire 'Q' remains suspended due to magnetic repulsion :-

(1) 33.3 mm (2) 0.33 mm (3) 333 mm (4) 3.33 mm

**9.** A long horizontal wire is rigidly fixed and carries 100A current. An another wire of linear mass density $2 \times 10^{-3}$ kg/m placed below and parallel to the fixed wire. If the free wire kept 2cm below and hangs in air, then current in free wire is :-

(1) 19.6 A (2) 9.8 A (3) 4.9 A (4) 100 A

# Magnetism and Matter

## Bar Magnet and Magnetic Dipole

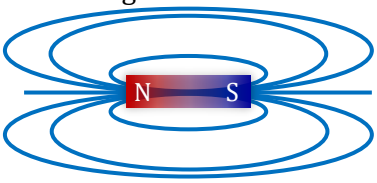
### Bar Magnet

A bar magnet is a rectangular piece of the object. It is made up of iron, steel or any other ferromagnetic substance or ferromagnetic composite, having permanent magnetic properties.

Two poles are present in a Bar magnet, North Pole and South Pole.

**Properties of bar magnets**

1. When a bar magnet is suspended freely near the surface of the earth, it tends to align always in the nearly North-South direction. One end of the bar magnet always pointing towards the geographic North is called the North pole of the magnet. The other end that always point towards the geographic South is called the South pole of the magnet.
2. Like poles repel and unlike poles attract each other.
3. The North and South poles of a bar magnet cannot be separated. The magnetic monopoles do not exist.
4. Bar magnets are made up of ferromagnetic materials, like iron, cobalt, nickel etc.
5. The magnetic field lines outside a bar magnet move from north pole to south pole.
6. The magnetic field lines inside a bar magnet move from south pole to north pole.




**Characteristics of Magnetic Field lines**

1. The tangent to a magnetic field line at a point gives the direction of the net magnetic field acting at that point
2. Magnetic field lines are always closed paths, whatever may be the source of magnetic field. Hence, there is no beginning or end point for a magnetic field line. There are no sources or sinks for magnetic field lines in a magnetic field. That is, there is no isolated magnetic pole.
3. The magnetic field lines can never intersect to each other.
4. The number of magnetic field lines across unit area in a region of space is a measure of the strength of the magnetic field in that region.

### Magnetic Dipole

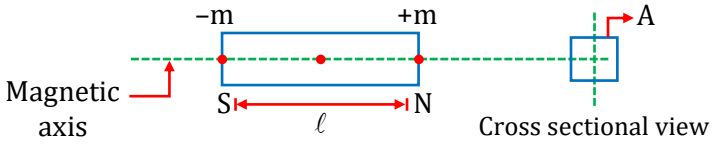
A magnetic dipole consists of a pair of magnetic poles of equal and opposite strength separated by small distance.

**Example:** Magnetic needle, Bar Magnet, Current carrying coil/solenoid etc.

Dipole Moment of a bar magnet

Dipole Moment $\rightarrow \vec{M}$

- Vector quantity
- Direction: South $\rightarrow$ North
- Unit: A-m$^2$




The magnetic dipole moment of a bar magnet is defined as a vector quantity having magnitude equal to the product of pole strength (m) with effective length ($\ell$) and directed along the axis of the magnet from south pole to north pole. $\vec{M} = m\vec{\ell}$

Where m : Pole Strength (Unit: A-m)

$\ell$ : effective distance between North pole and South pole

M : Dipole Moment of bar magnet

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**Note:**

- Number of field lines emerging from each pole indicate the pole strength of pole.
  Pole strength (m) of each pole $\propto$ area of cross section for a given bar magnet.
- As magnetic moment is a vector, in case of two magnets having magnetic moments $M_1$ and $M_2$ with angle $\theta$ between them, the resulting magnetic moment.

$$M=\left[M_1^2+M_2^2+2M_1M_2\cos\theta\right]^{1/2} \text{ with } \tan\phi=\left[\frac{M_2\sin\theta}{M_1+M_2\cos\theta}\right]$$

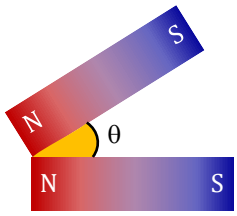
- Inverse square law (Coulomb law) : The magnetic force between two isolated magnetic poles of strength $m_1$ and $m_2$ lying at a distance 'r' is directly proportional to the product of pole strength and inversely proportional to the square of distance between their centres. The magnetic force between the poles can be attractive or repulsive according to the nature of the poles.

$$\left.\begin{array}{l}F_m\propto m_1m_2\\ F_m\propto\dfrac{1}{r^2}\end{array}\right]\ F_m=k\frac{m_1m_2}{r^2}\quad \text{where } k\begin{cases}\dfrac{\mu_0}{4\pi}\text{(S.I.)}\\ 1\text{(C.G.S.)}\end{cases}$$

Inverse square law of Coulomb in magnetism is applicable only for two long bar magnets because isolated poles cannot exist.

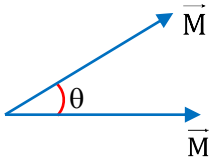
**Illustration 83:**

Dipole moment of a bar magnet is M. Find effective dipole moment of the given combination of two such magnets.




**Solution:**

Here




So, $M_{net}=2M\cos\frac{\theta}{2}$

**Illustration 84:**

The force between two magnetic poles in air is 9.604 mN. If one pole is 10 times stronger than the other, calculate the pole strength of each if distance between two poles is 0.1 m?

**Solution:**

Force between poles $F=\frac{\mu_0}{4\pi}\frac{m_1m_2}{r^2}$ or $9.604\times10^{-3}=\frac{10^{-7}\times m\times10m}{0.1\times0.1}$

or $m^2=96.04$ or $m=9.8$ N/T

So, strength of the other pole is $9.8\times10=98$ N/T

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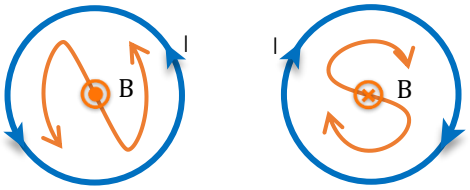
## Magnetic Moment of Current Carrying Coil (Loop)

Current carrying coil (or loop) behaves like magnetic dipole. The face of coil in which current appears to flow anticlockwise acts as north pole while face of coil in which current appears to flow clock wise acts as south pole.

A current carrying coil acts as a Magnetic Dipole




ACW → North Pole  CW → South Pole

- A loop of geometrical area 'A', carries a current 'I' then magnetic moment of coil M = IA
- A coil of turns 'N', geometrical area 'A', carries a current 'I' then magnetic moment M = NIA

  Magnetic moment of current carrying coil is an axial vector $\vec{M} = NI\vec{A}$ where $\vec{A}$ is a area vector perpendicular to the plane of the coil and along its axis. SI unit : A-m$^2$ or J/T

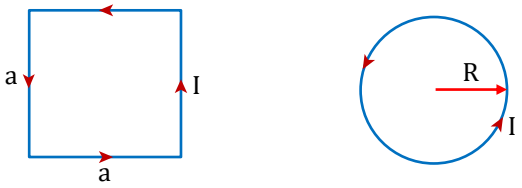
Direction of $\vec{M}$ find out by right hand thumb rule

- Curling fingers $\Rightarrow$ In the direction of current
- Thumb $\Rightarrow$ Gives the direction of $\vec{M}$

  For a current carrying coil, its magnetic moment and magnetic field vectors both are parallel axial vectors.

### Illustration 85:

Calculate Magnetic Moment of given loops.



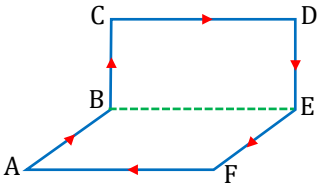

(A)  (B)

### Solution:

(A) M = IA = I($a^2$), (B)  M = IA = I($\pi R^2$)

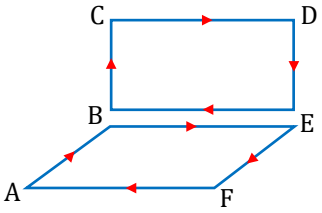
### Illustration 86:

Find the magnitude of magnetic moment of the current carrying loop ABCDEFA. Each side of the loop is 10 cm long and current in the loop is i = 2.0 A




### Solution:

By assuming two equal and opposite currents in BE, two current carrying loops (ABEFA and BCDEB) are formed. Their magnetic moments are equal in magnitude but perpendicular to each other.




Hence, $M_{net} = \sqrt{M^2 + M^2} = \sqrt{2}M$ where M = iA = (2.0)(0.1)(0.1) = 0.02 A-m$^2$

$M_{net} = (\sqrt{2})(0.02)$ A-m$^2$ = 0.028 A-m$^2$

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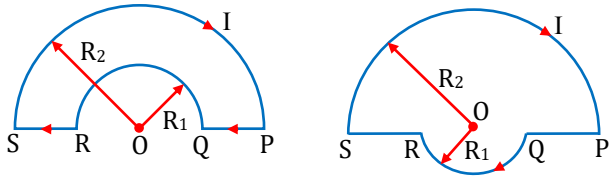
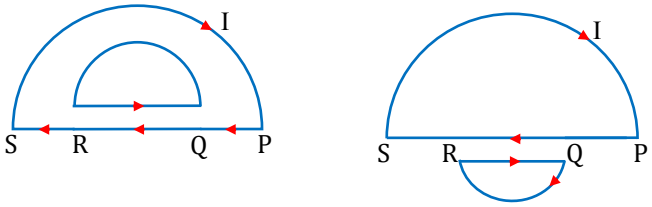
### Illustration 87:

The wire loop PQRSP formed by joining two semi-circular wires of radii $R_1$ and $R_2$ carries a current I as shown in fig. What is the magnetic moment of the loop in cases (A) and (B)?




### Solution:

By assuming two equal and opposite currents in QR two semi-circular current carrying loops are formed

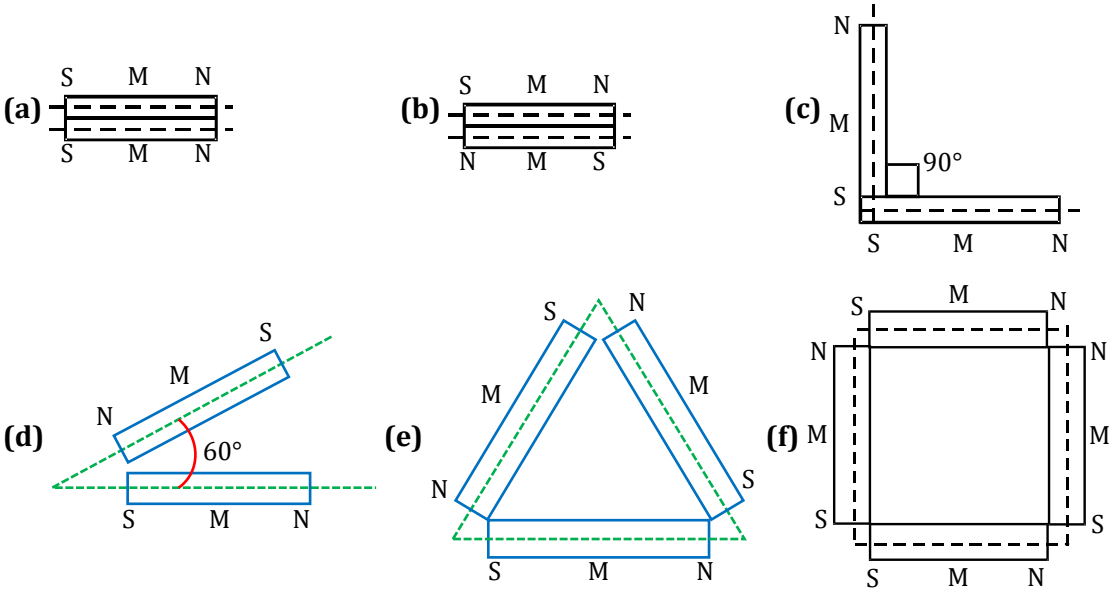



(A) $\vec{M}=NI\vec{A}=1\times I\left[\frac{1}{2}\pi R_2^2\otimes+\frac{1}{2}\pi R_1^2\odot\right]=\frac{1}{2}\pi I\left[R_2^2-R_1^2\right]\otimes$

(B) $\vec{M}=NI\vec{A}=1\times I\left[\frac{1}{2}\pi R_2^2\otimes+\frac{1}{2}\pi R_1^2\otimes\right]=\frac{1}{2}\pi I\left[R_2^2+R_1^2\right]\otimes$

## BEGINNER'S BOX-7

**1.** Two/three/four identical bar magnets of magnetic moment 'M' are combined according to figure. Find net magnetic moment of the system.




**2.** A bar magnet of magnetic moment 'M' is cuts into two equal parts in following two fashions (a) Parallel to its length (b) Perpendicular to its length, then what happens to the magnetic moment of each part.

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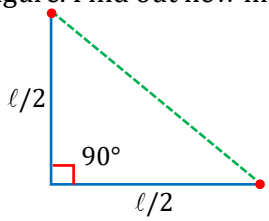
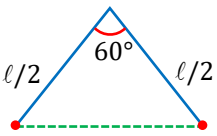
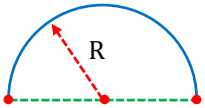
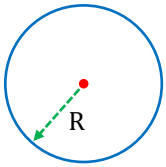
**3.** A straight steel wire of length '$\ell$' has a magnetic moment 'M'. It is bent into various shapes shown in figure. Find out new magnetic moment of each shape.

(a)



(b)


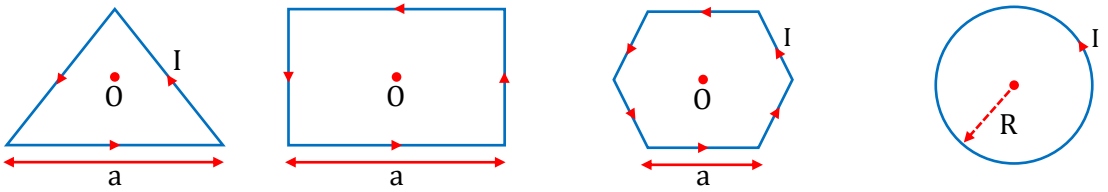

(c)



(d)



**4.** A square current carrying loop of side 'a' produced magnetic field at its centre is 'B'. Find magnetic moment of this loop in terms of 'B'.

**5.** A wire of length '$\ell$' carries a current bent into various regular polygons shown below. Find magnetic moment in terms of length of the wire.

(a) Equilateral triangle (b) Square (c) Regular hexagon (d) Circular loop




**6.** A current carrying wire of length '$\ell$' is bent to form a circular coil of one turn. After that it is again bent to form a circular coil of two turns. Find ratio of magnetic moment of coils respectively.

**7.** Circular coils of different turns are made by a constant length current carrying wire. Find number of turns in coil for its maximum magnetic moment.

**8.** Two identical current carrying coils are concentric and perpendicular to each other. If magnetic moment of each coil is 'M'. Find resultant magnetic moment of two coils.

## Torque and Force on Magnetic Dipole

### Torque on a Magnetic Dipole in Uniform Magnetic Field

When a bar magnet is placed in a uniform magnetic field the two poles experience a force. They are equal in magnitude and opposite in direction and do not have same line of action. They constitute a couple of forces which produces a torque. The torque tries to rotate the magnet so as to align it parallel too direction of field

**Torque due to force couple**

**(a) Bar magnet**

Consider a bar magnet of magnetic moment $\vec{M}$ held in a uniform magnetic field $\vec{B}$ making an angle $\theta$ with the field. Then the magnet experiences a torque given by

$\tau$ = force × perpendicular distance between force couple

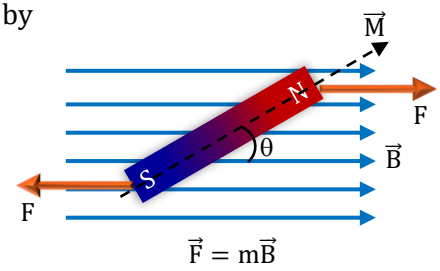
$\tau = F(\ell \sin\theta)$

$\tau = (mB)(\ell \sin\theta)$

$\tau = (m\ell)B\sin\theta$

$\tau = MB\sin\theta$, where $M = m\ell$

$\vec{\tau} = \vec{M} \times \vec{B}$




$\tau = MB\sin\theta$ → $\theta = 90° \Rightarrow \tau = MB$ (maximum)

→ $\theta = 0°$ or $180° \Rightarrow \tau = 0$ (minimum)

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**(b) Coil or Loop**

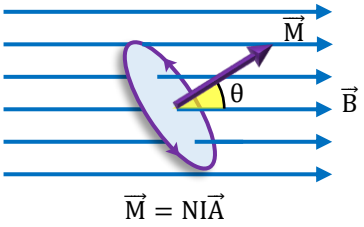
If a loop carries a current of magnitude I and magnetic moment $\vec{M}$ held in a uniform magnetic field $\vec{B}$ making an angle $\theta$

Then the loop experiences a torque given by

$\vec{\tau} = \vec{M} \times \vec{B}$

$\tau = MB\sin\theta$

$\vec{\tau} = I(\vec{A} \times \vec{B})$




If loop have N turns then

$\vec{\tau} = NI(\vec{A} \times \vec{B})$

$\tau = NIAB\sin\theta$

$\tau = NIAB\sin\theta$ → $\theta = 90°$ $\Rightarrow$ $\tau = NIAB$ (maximum)

→ $\theta = 0°$ or $180°$ $\Rightarrow$ $\tau = 0$ (minimum)

**Note:**

- Torque on dipole is an axial vector and it is directed along axis of rotation of dipole.
- Tendency of torque on dipole is try to align the $\vec{M}$ in the direction of $\vec{B}$ or tries to makes the axis of dipole parallel to $\vec{B}$ or makes the plane of coil (or loop) perpendicular to $\vec{B}$.
- Dipole in uniform magnetic field → $F_{net} = 0$ (no translatory motion)
  → $\tau$ may or may not be zero (decides by $\theta$)
- Dipole in non-uniform magnetic field → $F_{net}$ may or may not be zero
  → $\tau$ may or may not be zero (decides by $\theta$)

### Illustration 88:

A magnet of magnetic moment $50\hat{i}$ A–m$^2$ placed along x–axis. Where magnetic field is $\vec{B} = (0.5\,\hat{i} + 3.0\,\hat{j})$ tesla. The torque acting on magnet is : –

### Solution:

$$\vec{\tau} = \vec{M} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 50 & 0 & 0 \\ 0.5 & 3 & 0 \end{vmatrix} = (0)\,\hat{i} - (0)\,\hat{j} + (150)\,\hat{k} \text{ N–m} = 150\,\hat{k} \text{ N–m}$$

### Illustration 89:

A bar magnet of length 10 cm and having the pole strength equal to $10^{-3}$ Wb is kept in a magnetic field having magnetic induction (B) equal to $4\pi \times 10^{-3}$ T. It makes an angle of 30° with the direction of magnetic induction. The value of the torque acting on the magnet is

### Solution:

Torque, $\tau = MB\sin\theta = 0.1 \times 10^{-3} \times 4\pi \times 10^{-3} \times \sin 30° = 10^{-7} \times 4\pi \times \frac{1}{2} = 2\pi \times 10^{-7}$ Nm

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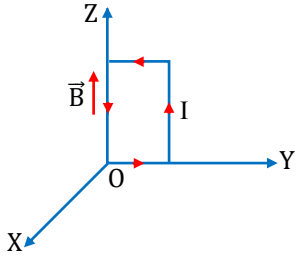
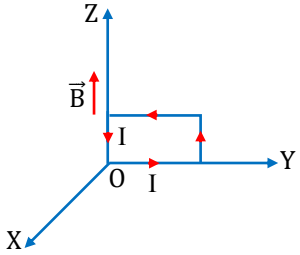
### Illustration 90:

A uniform magnetic field of 5000 gauss is established along the positive z-direction. A rectangular loop of side 20 cm and 5 cm carries a current of 10 A is suspended in this magnetic field. What is the torque on the loop in the different cases shown in the following figures? What is the force in each case? Which case corresponds to stable equilibrium?

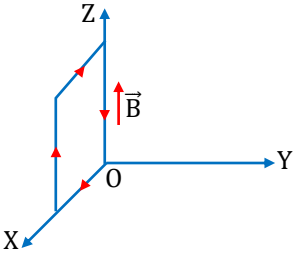
(a)



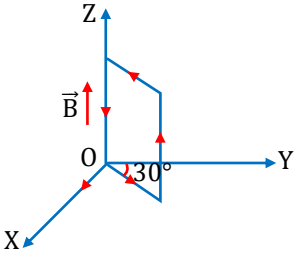

(b)



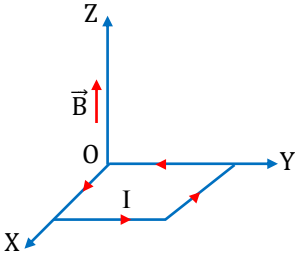

(c)




(d)




(e)




### Solution:

(a) Torque on loop, $\tau = BIA\sin\theta$

Here, $\theta = 90°$; B=5000 gauss = $5000 \times 10^{-4}$ tesla = 0.5 tesla

I = 10 ampere, A = $20 \times 5\text{cm}^2 = 100 \times 10^{-4} = 10^{-2}\ \text{m}^2$

Now, $\tau = 0.5 \times 10 \times 10^{-2} = 5 \times 10^{-2}$ Nm  It is directed along –Y axis

(b) Same as (a).

(c) $\tau = 5 \times 10^{-2}$ Nm along –x-direction

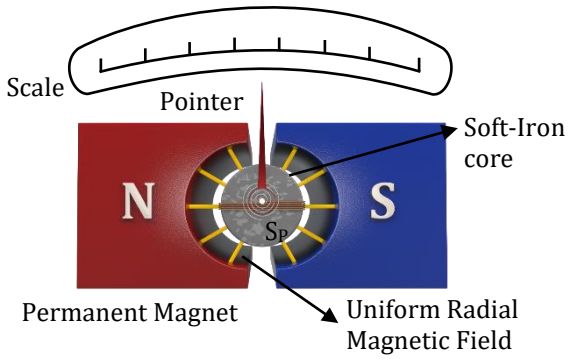
(d) $\tau = 5 \times 10^{-2}$ N m at an angle of 240° with +x direction.

(e) $\tau$ is zero. [$\because$ Angle between plane of loop and direction of magnetic field is 90°]

Resultant force is zero in each case. Case (e) corresponds to stable equilibrium.

## Moving Coil Galvanometer

**Construction**




The galvanometer consists of a coil, with many turns, free to rotate about a fixed axis, in a uniform radial magnetic field. There is a cylindrical soft iron core which not only makes the field radial but also increases the strength of the magnetic field. A spiral spring is also attached to the coil which resists the rotation of coil.

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**Working Principle**

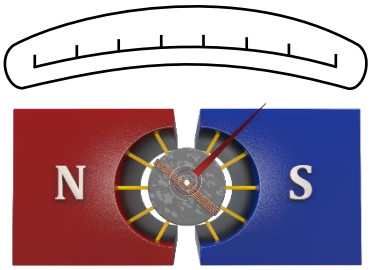
When a current carrying coil is placed in a magnetic field, it experiences a torque

$\vec{\tau} = NI(\vec{A} \times \vec{B})$

$\tau = NIAB\sin\theta$

$\tau = NIAB$ ($\theta = 90°$due to radial magnetic field)

In this case I=current through coil,
A = Area of coil,
B = Magnetic field
N = no. of turns,




This torque tends to rotate the coil by $\phi$ angle.

The spring S provides a counter torque $C\phi$ that balances the magnetic torque. In equilibrium,

$$C\phi = NIAB \quad \Rightarrow \quad I = \frac{C}{NAB}\phi$$

$I \propto \phi$

Hence, it means the deflection produced is proportional to the current flowing through the galvanometer.

**Current Sensitivity**

Current sensitivity is defined as the deflection produced in the galvanometer when a unit current flows through it.

$$C.S. = \frac{\phi}{I} = \frac{NAB}{C}(\text{rad}/\text{A})$$

**Voltage Sensitivity**

Voltage sensitivity is equal to the deflection per unit voltage applied across voltmeter.

$$V.S. = \frac{\phi}{V} = \frac{\phi}{IR} = \frac{NAB}{CR}(\text{rad}/\text{V})$$

**Advantage of a Moving Coil Galvanometer**

1. As the deflection of the coil is proportional to the current passed through it, so a linear scale can be used to measure the deflection.
2. A moving coil galvanometer can be made highly sensitive by increasing N, B, A and decreasing C.
3. As the coil is placed in a strong magnetic field of a powerful magnet, its deflection is not affected by external magnetic fields. This enables us to use the galvanometer in any position.
4. As the coil is wound over a metallic frame, the eddy currents produced in the frame bring the coil to rest quickly.

**Disadvantage of a Moving Coil Galvanometer**

1. The main disadvantage is that its sensitivity cannot be changed at will.
2. All types of moving coil galvanometer are easily damaged by overloading. A current greater than that which the instrument is intended to measure will burn out its hair springs or suspension.

**Illustration 91:**

A moving coil galvanometer has a coil with 175 turns and area $1cm^2$. It uses a torsion band of torsion constant $10^{-6}$ N-m/rad. The coil is placed in a magnetic field B parallel to its plane. The coil deflects by 1° for a current of 1 mA. The value of B(in Tesla) is approximately.

**Solution:**

$\tau = \vec{M} \times \vec{B}$

$C\phi = INAB$

$$10^{-6} \times \frac{\pi}{180} = 10^{-3} \times 10^{-4} \times 175 \times B$$

$B = 10^{-3}$ T.

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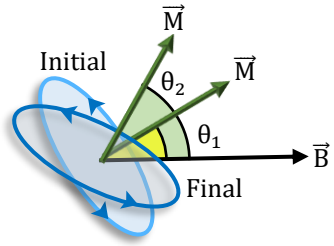
## Potential Energy of Magnetic Dipole

### Work Done in Rotating the Coil in Uniform Magnetic Field

When a bar magnet or coil of dipole moment M is kept in a uniform magnetic field B it experiences a torque which tries to align it in the direction of external magnetic field $\vec{B}$. Work has to be done in rotation the magnet or coil.




The work done in rotating dipole by small-angle dθ is $dW = \tau d\theta$

The work done in rotating the magnet or coil from $\theta = \theta_1$ to $\theta = \theta_2$ is given by

$$W = \int_{\theta_1}^{\theta_2} \tau d\theta$$

$$W = \int_{\theta_1}^{\theta_2} MB\sin\theta\, d\theta$$

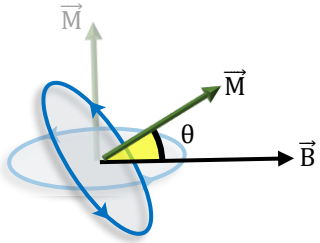
$$W = MB\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$W = -MB(\cos\theta_2 - \cos\theta_1)$$

$$W = MB(\cos\theta_1 - \cos\theta_2)$$

### Potential Energy of the Coil in Uniform Magnetic Field

The potential energy of dipole defined as work done in rotating the dipole from a direction perpendicular to the given direction.



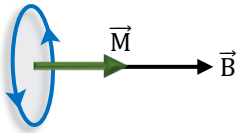
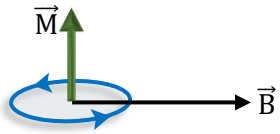
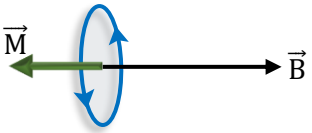

Work done in rotating the dipole from $\theta_1 = 90°$ to $\theta_2 = 0°$

$$W = MB(\cos 90^o - \cos\theta)$$

$$U = -MB\cos\theta$$

In vector form $\boxed{U = -\vec{M}.\vec{B}}$

**Important Note**

| Case-I | Case-II | Case-III |
|---|---|---|
|  |  |  |
| When $\vec{M}$ and $\vec{B}$ are parallel<br>$\theta = 0°$<br>$\tau = 0$<br>$U_{min} = -MB$<br>The dipole has minimum potential energy and it is in stable equilibrium. | When $\vec{M}$ and $\vec{B}$ are perpendicular<br>$\theta = 90°$<br>$\tau = MB$<br>$U = 0$<br>No Equilibrium as $\tau \neq 0$ | When $\vec{M}$ and $\vec{B}$ are anti-parallel<br>$\theta = 180°$<br>$\tau = 0$<br>$U_{max} = MB$<br>The dipole has maximum potential energy and it is in unstable equilibrium. |

### Illustration 92:

A circular coil of 100 turns and having a radius of 0.05 m carries a current of 0.1 A. Calculate the work required to turn the coil in an external field of 1.5 T through 180° about an axis perpendicular to the magnetic field? The plane of coil is initially at right angles to magnetic field.

### Solution:

Work done $W = MB(\cos\theta_1 - \cos\theta_2) = NIAB(\cos\theta_1 - \cos\theta_2)$

$\Rightarrow W = NI\pi r^2B(\cos\theta_1 - \cos\theta_2) = 100 \times 0.1 \times 3.14 \times (0.05)^2 \times 1.5(\cos 0° - \cos\pi) = 0.2355J$

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**Illustration 93:**

A bar magnet of magnetic moment 1.5 $JT^{-1}$ lies aligned with the direction of a uniform magnetic field of 0.22 T. What is the amount of work required to turn the magnet so as to align its magnetic moment.

(i) Normal to the field direction? (ii) Opposite to the field direction?

**Solution:**

Here, $M = 1.5\ JT^{-1}$, $B = 0.22$ T.

P.E. with magnetic moment aligned to field = – MB

P.E. with magnetic moment normal to field = 0

P.E. with magnetic moment antiparallel to field = + MB

(i) Work done = increase in P.E. = 0 – (–MB) = MB = 1.5 × 0.22 = 0.33 J.

(ii) Work done = increase in P.E. = MB – (–MB) = 2MB = 2 × 1.5 × 0.22 = 0.66 J.

**Illustration 94:**

A short bar magnet of magnetic moment 0.32 J/T is placed in uniform field of 0.15 T. If the bar is free to rotate in plane of field then which orientation would correspond to its (i) stable and (ii) unstable equilibrium? What is potential energy of magnet in each case?

**Solution:**

(i) If M is parallel to B then $\theta = 0°$. So potential energy $U = U_{min} = -MB$

$U_{min} = -MB = -0.32 \times 0.15\ J = -4.8 \times 10^{-2}$ J (stable equilibrium)

(ii) If M is antiparallel to B then $\theta = \pi$ So potential energy

$U = U_{max} = +MB = +0.32 \times 0.15 = 4.8 \times 10^{-2}$ J (unstable equilibrium.)

## BEGINNER'S BOX-8

**1.** When a magnetic needle placed in non uniform field the needle may experiences a :-

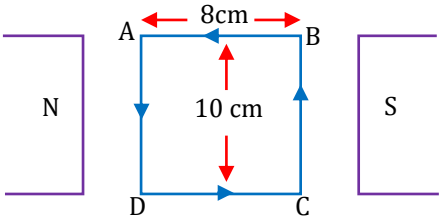
(1) Force (2) Torque

(3) Force and torque (4) No force and no torque

**2.** A magnet of magnetic moment $50\hat{i}$ A–m² placed along x–axis. Where magnetic field is $\vec{B} = (0.5\hat{i} + 3.0\hat{j})$ tesla. The torque acting on magnet is :–

(1) $175\hat{k}$ N–m (2) $150\hat{k}$ N–m (3) $75\hat{k}$ N–m (4) $25\sqrt{37}\hat{k}$ N–m

**3.** A coil of 100 turns kept in magnetic field B = 0.2T, carries a current of 2A as shown in figure. Torque on coil and which side of coil comes out from the plane of the paper :-




(1) 0.16 N-m, BC side comes out from plane of paper

(2) 0.16 N–m, AD side comes out from plane of paper

(3) 0.32 N, BC side comes out from plane of paper

(4) 0.32 N–m, AD side comes out from plane of paper

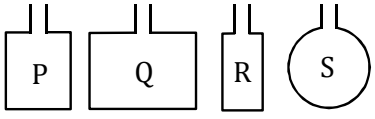
**4.** Four wires each of length 2.0 m, are bent in four loop P,Q,R and S then suspended in uniform magnetic field. All loop carries equal currents, which statement of the following is true :

(1) Couple acting on the loop P is greatest.

(2) Couple acting on the loop Q is greatest.

(3) Couple acting on the loop R is greatest

(4) Couple acting on the loop S is greatest.

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**5.** A current carrying wire is bent to form a circular coil. If this coil is placed in any other magnetic field then for maximum torque on the coil, the number of turns will be -

(1) 1 (2) 2 (3) 4 (4) 8

**6.** A magnet of magnetic moment 4A–m$^2$ is held in a uniform magnetic field $5 \times 10^{-4}$T with the magnetic moment vector makes an angle 30° with the field. Work done in increasing the angle from 30° to 45°:-

(1) $3.2 \times 10^{-4}$ J (2) $1.6 \times 10^{-4}$ J (3) $1.6 \times 10^{-3}$ J (4) $3.2 \times 10^{-3}$ J

**7.** A magnetic needle laying parallel to the magnetic field requires W units of work to turn it through 60°. Torque needed to maintain the needle in this position :-

(1) W (2) $\sqrt{3}$ W (3) $\frac{W}{2}$ (4) $\frac{\sqrt{3}W}{2}$

**8.** A bar magnet has a magnetic moment 2.5 A–m$^2$ and it is placed in a magnetic field of 0.2T. Calculate the Work done in turning the magnet from parallel to antiparallel position relative to the field direction:-

(1) 0.1 J (2) 1J (3) 2J (4) 5J

**9.** A circular coil of 'N' turns, 'R' radius carries a 'i' current. Work done in rotating this coil in an external magnetic field from $\theta_1 = 0°$ to $\theta_2 = 90°$ :-

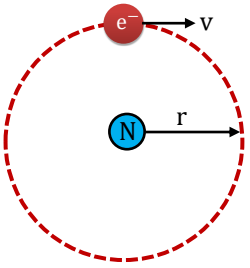
(1) $2\pi NiR^2B$ (2) $\frac{\pi NiR^2B}{2}$ (3) $\pi NiR^2B$ (4) $\frac{\sqrt{3}\pi NiR^2B}{2}$

**10.** A short bar magnet placed with its axis at 30° with uniform magnetic field of 0.16T, experiences a torque of magnitude 0.032 N–m. The potential energy of bar magnet in stable equilibrium and in unstable equilibrium respectively :-

(1) –0.064 J, + 0.064 J (2) –0.032 J, + 0.032 J
(3) –0.016 J, + 0.016 J (4) zero, zero

## Atomic Magnetism

Consider an electron revolving around a nucleus in an atom. Due to motion of electron, orbit behaves like a current carrying loop. Due to this, magnetic field is produced at center of orbit. This is called 'atomic magnetism'.




**(a) Effective current**

$$I = \frac{q}{T} = \frac{e}{T} = ef = \frac{e\omega}{2\pi} = \frac{ev}{2\pi r}$$

$$I = \frac{qv}{2\pi r} \text{ (In general)}$$

**(b) Induced Magnetic Field at center**

$$B_{in} = \frac{\mu_o I}{2r} = \frac{\mu_0 e}{2Tr} = \frac{\mu_0 ef}{2r} = \frac{\mu_0}{2r}\left(\frac{ev}{2\pi r}\right) = \frac{\mu_0 ev}{4\pi r^2}$$

$$B_{in} = \frac{\mu_0 qv}{4\pi r^2} \text{ (In general)}$$

**(c) Magnetic Moment**

$$M = IA = \left(\frac{ev}{2\pi r}\right)\pi r^2 = \frac{e\omega r^2}{2} = \frac{evr}{2}$$

$$M = \frac{qvr}{2} \text{ (In general)}$$

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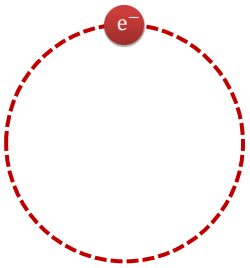
- **Relation between magnetic moment and angular momentum of orbital electron**

Angular Momentum of electron

$L = mvr$




Relation between $\vec{L}$ and $\vec{M}$

$$M = \frac{evr}{2}$$

$$\Rightarrow \frac{M}{L} = \frac{\left(\frac{evr}{2}\right)}{mvr} = \frac{e}{2m}$$

$\frac{M}{L} = \frac{q}{2m}$ (In general)

**Bohr magneton ($\mu_B$)**

According of Bohr's theory, angular momentum of orbital electron is given by

$L = \frac{nh}{2\pi}$, where n = 1, 2, 3 ..... and h is plank's constant.

As per Bohr's Theory

$$L = \frac{nh}{2\pi}$$

$$\frac{M}{L} = \frac{e}{2m}, M = \frac{e}{2m}L$$

$$M = \frac{neh}{4\pi m}$$

- If n = 1 then $M = \frac{eh}{4\pi m}$, which is Bohr magneton denoted by $\mu_B$
- Definition of $\mu_B$ : Bohr magneton can be defined as the magnetic moment of orbital electron which revolves in first orbit of an atom.

$$\mu_B = \frac{eh}{4\pi m} = \frac{1.6\times10^{-19}\times6.6\times10^{-34}}{4\times3.14\times9.1\times10^{-31}} = 0.923 \times 10^{-23}\ Am^2$$

**Basic elements of atomic magnetism for first orbit of H-atom (n = 1, Z = 1)**

(a) Accurate form :- ($v = 2.18 \times 10^6$ m/sec, $f = 6.6 \times 10^{15}$ cy/sec. r = 0.529Å)

- Orbital current I = 0.96 mA
- Magnetic induction at nucleus position $B_N$ = 12.8 T
- Magnetic moment of orbital electron $M = 0.923 \times 10^{-23}\ Am^2$

(b) Simple form :- ($v \simeq 2 \times 10^6$ m/sec, $f \simeq 6 \times 10^{15}$cy/sec, $r \simeq 0.5$Å)

- Orbital current I = 1mA
- Magnetic induction at nucleus position $B_N \simeq 4\pi$T
- Magnetic moment of orbital electron $M = \mu_B\ Am^2$

## Magnetic moment of rigid bodies

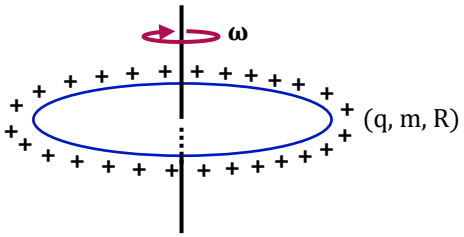
A nonconducting charged body is rotated with some angular speed :-

In this case the ratio of magnetic moment and angular momentum is constant which is equal to $\frac{q}{2m}$

here q = charge and m = the mass of the body.

**Note:**

In case of a ring, of mass m, radius R and charge q distributed on it circumference.




Angular momentum, $L = I'\omega = (mR^2)(\omega)$ ...(i)

[here I' = Moment of Inertia]

Magnetic moment, $M = iA = (qf)(\pi R^2)$

$$M = (q)\left(\frac{\omega}{2\pi}\right)(\pi R^2) = q\frac{\omega R^2}{2} \quad \text{...(ii)}$$

$\because f = \frac{\omega}{2\pi}$ From Equation (i) and (ii) $\frac{M}{L} = \frac{q}{2m}$

Although this expression is derived for simple case of a ring, it holds good for other bodies also. For example, for a disc or a sphere. $M = \frac{qL}{2m} \Rightarrow M = \frac{q(I'\omega)}{2m}$, where $L = I'\omega$

**Example :**

| R.B. Name | Moment of Inertia (I') | $M = \frac{q}{2m}(I'\omega)$ |
|---|---|---|
| Ring | $mR^2$ | $\frac{1}{2}q\omega R^2$ |
| Disc | $\frac{mR^2}{2}$ | $\frac{1}{4}q\omega R^2$ |
| Solid sphere | $\frac{2}{5}mR^2$ | $\frac{1}{5}q\omega R^2$ |
| Hollow sphere (Or spherical shell) | $\frac{2}{3}mR^2$ | $\frac{1}{3}q\omega R^2$ |

## BEGINNER'S BOX-9

**1.** Magnetic moment and angular momentum of an orbital electron are 'M' and L respectively. Specific charge of orbital electron :-

(1) $\frac{M}{2L}$ (2) $\frac{2M}{L}$ (3) $\frac{L}{2M}$ (4) $\frac{2L}{M}$

**2.** A particle of charge 'q' and mass 'm' is moving in a circular orbit with angular speed 'ω'. The ratio of its magnetic moment to that of its angular momentum depends on :-

(1) ω, q (2) ω, q, m (3) q,m (4) ω,m

**3.** Magnetic moment of orbital electron in first orbit is $\mu_B$ then magnetic moment of that electron in third orbit :-

(1) $\mu_B$ (2) $\frac{\mu_B}{3}$ (3) $3\mu_B$ (4) $9\mu_B$

**4.** A helium nucleus is moving in a circular path of radius 0.8m. If it takes 2 sec to complete one revolution. Magnetic field produced at the centre of the circle :-

(1) $\mu_0 \times 10^{-19}$ T (2) $\frac{10^{-19}}{\mu_0}$ T (3) $2\times 10^{-19}$ T (4) $\frac{2\times 10^{-19}}{\mu_0}$ T

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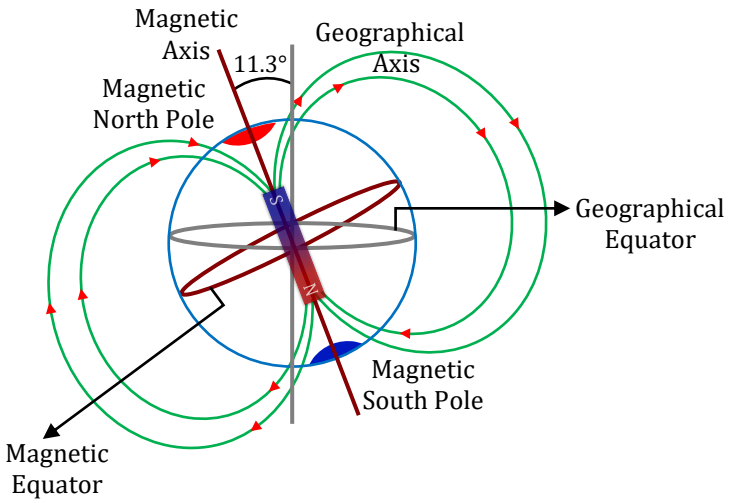
## Geo-Magnetism

The branch of physics which deals with the study of earth's magnetic field is called geomagnetism.

The idea that earth is magnetised was first suggested towards the end of the sixteenth century by Dr. William Gilbert. The origin of earth's magnetism is still a matter of conjecture among scientists but it is agreed upon that the earth behaves as a magnetic dipole inclined at a small angle (11.3°) to the earth's axis of rotation with its south pole pointing north.

### Important Terms




**(1) Magnetic South Pole**

Point on Earth where Earth's Magnetic Field lines emerge.

**(2) Magnetic North Pole**

Point on Earth where Earth's Magnetic Field lines end.

**(3) Geographic axis**

It is a straight line which passes through the geographical poles of the earth. It is also called axis of rotation or polar axis of the earth.

**(4) Magnetic Axis**

Axis passing through Earth's Magnetic Poles.

**(5) Geographic equator**

It is a great circle on the surface of the earth, in a plane perpendicular to the geographic axis. All the points on the geographic equator are at equal distance from the geographic poles.

A great plane which passes through geographic equator and perpendicular to the geographic axis called geographic equatorial plane. This plane cuts the earth in two equal parts, a part has geographic north called northern hemisphere (NHS) and another part has geographic south called southern hemisphere (SHS).

**(6) Magnetic Equator**

It is a great circle on the surface of the earth in a plane perpendicular to the magnetic axis. All the point on the magnetic equator are at equal distance from magnetic poles.

**(7) Geographical Meridian**

It is a plane passing through geographical axis, and number of geographical meridian is ∞.

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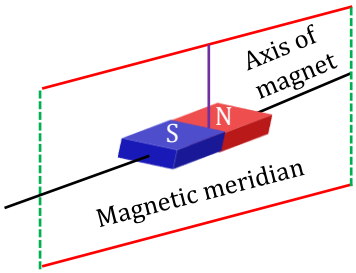
**(8) Magnetic Meridian**

(i) It is a vertical plane which passes through magnetic axis of the earth.

(ii) At a particular place it is a vertical plane, which passes through axis of free suspended bar magnet or magnetic needle.

(iii) At a particular place it is a vertical plane, which contains all the field lines of earth's magnetic field.




**Significance of Magnetic Meridian**

A magnetic needle which is free to swing always lies in the Magnetic Meridian.

### Illustration 95:

Lines of forces of earth's magnetic field are from

### Solution:

The magnetic field lines of Earth start from the north pole and enter the south pole.

Therefore, the magnetic field lines of Earth create loops from the centre of Earth that enter the south pole and exit in the north pole.

### Illustration 96:

Answer the following questions regarding earth's magnetism:

(a) If you made a map of magnetic field lines at Melbourne in Australia. would the lines seem to go into the ground or come out of the ground?

(b) A compass needle free to move in horizontal plane. In which direction it would stay, if located right on the geomagnetic north or south pole?

(c) The earth's magnetic field varies from point to point in space. Does it also change with time? If so, on what time scale does it change appreciably?

(d) The earth may have even reversed the direction of its field several times during its history of 4 to 5 billion years. How can geologists know about the earth's field in such distant past?

### Solution:

(a) Field lines of B due to the earth's magnetism would seem to come out of the ground.

(b) It will remain in any direction in horizontal plane.

(c) Yes, it does change with time. Time scale for appreciable change is roughly a few hundred years. But even on a much smaller scale of a few years, its variations are not completely negligible.

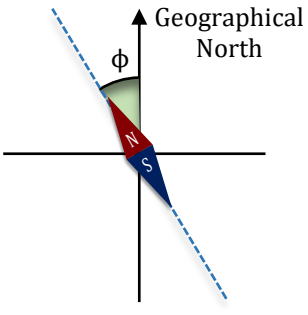
(d) Earth's magnetic field gets weakly 'recorded' in certain rocks during solidification. Analysis of this rock magnetism offers clues to geomagnetic history.

## Angle of Declination

**Angle of Declination (ϕ)**

(1) At a given place it is the angle between the geographical north-south direction and the direction indicated by a magnetic compass needle in its equilibrium.

(2) At a given place the acute angle between geographic meridian and the magnetic meridian is called angle of declination.



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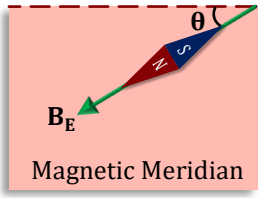
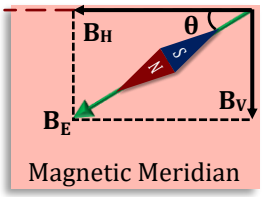
## Angle of Dip (θ)




- At a given place, it is the angle which the direction of resultant intensity of earth's magnetic field subtends with horizontal line in magnetic meridian.
- It is the angle made by a freely suspended magnetic needle (free to move in vertical plane) with the horizontal.
- Measured with help of an instrument: Dip Circle

## Horizontal component of earth's magnetic field ($B_H$)




At a given place horizontal component of earth magnetic field is the component of resultant magnetic field of the earth along the horizontal line in magnetic meridian.

$B_H = B\cos\theta$ and $B_v = B\sin\theta$ ...(1)

so that $\tan\theta = \frac{B_V}{B_H}$ and $B = \sqrt{B_H^2 + B_V^2}$ ...(2)

**Note :** At magnetic poles $\Rightarrow \theta = 90°$ $\begin{cases} B_H = 0(\text{min}) \\ B_V = B(\text{max}) \text{ (fully)} \end{cases}$

At magnetic equator $\Rightarrow \theta = 0°$ $\begin{cases} B_H = B(\text{max}) \text{ (fully)} \\ B_V = 0(\text{min}) \end{cases}$

### Illustration 97:

At a place, the earth's horizontal component of magnetic field is $0.36 \times 10^{-4}$ Wb/m$^2$. If the angle of dip at that place is 60°, then the vertical component of earth's field at that place in Wb/m$^2$. will be approximately

### Solution:

From the relation, $B_H = B\cos\phi$ and $B_v = B\sin\phi$ $\quad \frac{B_v}{B_H} = \tan\phi$ or $B_v = B_H\tan\phi$

$$B_v = 0.36 \times 10^{-4} \times \tan 60° = 0.623 \times 10^{-4} \text{ Wb/m}^2$$

### Illustration 98:

The earth's magnetic field at a certain place has a horizontal component 0.3 gauss and the total strength 0.5 gauss. The angle of dip is

### Solution:

$$B^2 = B_v^2 + B_H^2 \Rightarrow B_v = \sqrt{B^2 - B_H^2} = \sqrt{(0.5)^2 - (0.3)^2} = 0.4$$

$$\text{Now, } \tan\phi = \frac{B_v}{B_H} = \frac{0.4}{0.3} = \frac{4}{3} \Rightarrow \phi = \tan^{-1}\left(\frac{4}{3}\right)$$

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**Illustration 99:**

If the angles of dip at two places are 30° and 45° respectively, then the ratio of horizontal components of earth's magnetic field at the two places will be.

**Solution:**

The horizontal components are $(B_H)_1 = B\cos\phi_1$ and $(B_H)_2 = B\cos\phi_2$

$$\therefore \quad \frac{(B_H)_1}{(B_H)_2} = \frac{\cos\phi_1}{\cos\phi_2} = \frac{\cos 30°}{\cos 45°} = \frac{\sqrt{3}}{2} \times \sqrt{2} = \sqrt{\frac{3}{2}}$$

**Illustration 100:**

Density of cosmic particles (positive charge) is more?

(1) Poles (2) Equator

**Solution:**

Density of cosmic particles (positive charge) is more at poles. The earth's magnetic field is strongest at the poles hence cosmic charges particles are deflected away from the equator.

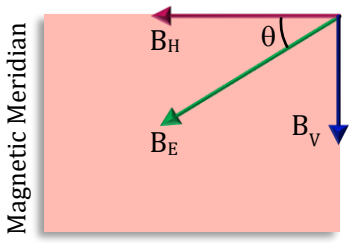
**Apparent Angle of Dip ($\theta'$)**

- When Magnetic needle is free to rotate only in a vertical plane other than the magnetic meridian, its angle with the horizontal is different than actual angle of dip.
- If the dip circle is not kept in the magnetic meridian, the needle will not show the correct direction of earth's magnetic field.
- The angle made by the needle with horizontal is called the apparent dip for this plane.

If dip circle lies in the plane of magnetic meridian then

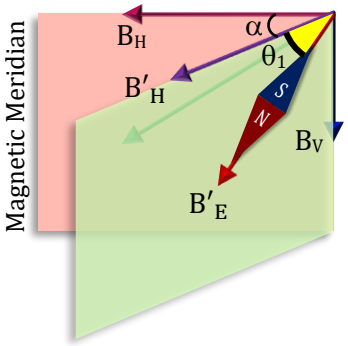
$$\tan\theta = \frac{B_V}{B_H}$$

$\theta$ is true angle of dip (In the plane of magnetic meridian)




If dip circle is at an angle $\alpha$ with magnetic meridian, the effective horizontal component in this plane is $B'_H = B_H\cos\alpha$

So, $\tan\theta_1 = \frac{B_V}{B'_H} = \frac{B_V}{B_H\cos\alpha} = \frac{\tan\theta}{\cos\alpha}$




$\theta_1$ is apparent angle of dip (In the plane other than magnetic meridian)

Apparent dip angle is always greater than actual dip angle.

Now if the dip circle is rotated through an angle 90° for this position. It will make angle (90 - $\alpha$) with the meridian, lets this time apparent dip is $\theta_2$

$$\text{Thus } \tan\theta_2 = \frac{\tan\theta}{\cos(90-\alpha)} = \frac{\tan\theta}{\sin\alpha} \qquad \text{...(ii)}$$

**Note:** As $\sin^2\alpha + \cos^2\alpha = 1$

$\cot^2\theta_1 + \cot^2\theta_2 = \cot^2\theta$

Thus, one can get the true dip angle $\theta$, without locating the magnetic meridian.

If apparent dip angles in two vertical planes which are at 90° from each other are $\theta_1$ and $\theta_2$ then actual dip angle $\theta$ is given by :-

$\cot^2\theta = \cot^2\theta_1 + \cot^2\theta_2$

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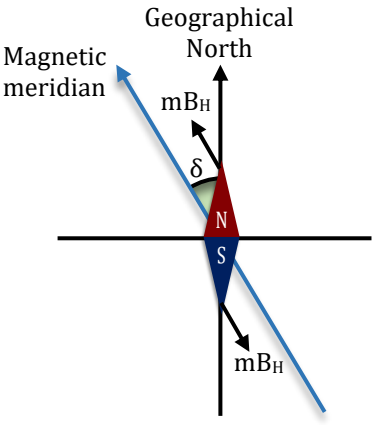
### Illustration 101:

A compass needle whose magnetic moment is 60 A $m^2$ pointing geographical north at a certain place, where the horizontal component of earth's magnetic field is 40μ Wb/$m^2$, experiences a torque 1.2 × $10^{-3}$ Nm. What is the declination at this place?




**Solution:**

As the compass needle is free to rotate in a horizontal plane and points along the magnetic meridian, so when it is pointing along the geographic meridian, it will experience a torque due to the horizontal component of earth's magnetic field

$\tau = MB_H\sin\theta$ where $\theta$ = angle between geographical and magnetic meridians called angle of declination

So, $\sin\theta = \dfrac{1.2\times10^{-3}}{60\times40\times10^{-6}} = \dfrac{1}{2} \Rightarrow \theta = 30°$

### Illustration 102:

If apparent dip angles in two vertical planes situated at complementary angles from magnetic meridian are $\cot^{-1}(3/5)$, and $\cot^{-1}(4/5)$, then true dip angle at that place is.

**Solution:**

If apparent dip angles in two vertical planes which are at 90º from each other are $\theta_1$ and $\theta_2$ then actual dip angle $\theta$ is given by :-

$\cot^2\theta = \cot^2\theta_1 + \cot^2\theta_2$ ...(i)

Given $\cot\theta_1 = \dfrac{3}{5}$ and $\cot\theta_2 = \dfrac{4}{5}$

$\cot^2\theta = \left(\dfrac{3}{5}\right)^2 + \left(\dfrac{4}{5}\right)^2 = \dfrac{9}{25} + \dfrac{16}{25} = 1$

$\cot^2\theta = 1$

$\theta = \cot^{-1}(1)$

$\theta = 45°$

## Application of geomagnetism

### Tangent Galvanometer

It is an instrument which can detect/measure electric d.c. currents.

It is also called moving magnet type galvanometer.

**Principle :-** It is based on 'tangent law'.

**Construction :-**

(i) It consists of a circular coil of a large number of turns of insulated copper wire wound over a vertical circular frame.
Plane of coil is in Magnetic meridian for measurement of current.

(ii) A small magnetic compass needle is pivoted at the centre of vertical circular coil. This needle can rotate freely in a horizontal plane.

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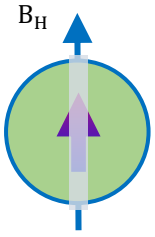
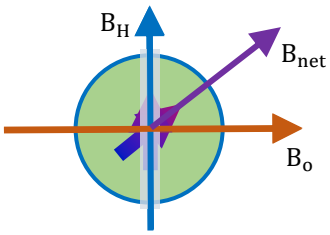
**Working :-**




- When there is no current in the coil, compass needle heads North (along $B_H$)
- **Tangent law :-** If a current is passed through the vertical coil, then magnetic field produced at its centre is perpendicular to the horizontal component of earth's magnetic field since coil is in magnetic meridian. So in the effect of two crossed fields ($B_H \perp B_0$) compass needle comes in equilibrium according to tangent law.




Due to resultant of earth horizontal component of magnetic field and coil, the magnetic needle deflects from its initial direction in the direction of $B_{net}$.

Now, torque on magnetic needle due to $B_H$ and $B_o$ become equal under equilibrium condition.

$\tau_1 = \tau_2$

$MB_H \sin\theta = MB_o \sin(90° - \theta)$

$B_H \sin\theta = B_o \cos\theta$

$B_0 = B_H \dfrac{\sin\theta}{\cos\theta} \Rightarrow \boxed{B_0 = B_H \tan\theta}$

$B_0 = B_H \tan\theta \Rightarrow \dfrac{\mu_0 NI}{2R} = B_H \tan\theta$

$I = \left(\dfrac{2B_H R}{\mu_0 N}\right) \tan\theta$

$I = K \tan\theta$, where $K = \dfrac{2B_H R}{\mu_0 N}$ called reduction factor of tangent galvanometer.

So, the electric current is proportional to the tangent of the angle of deflection i.e. $I \propto \tan\theta$

**Note:**

1. Tangent Galvanometer is also used to find $B_H$
2. The reduction factor of a tangent galvanometer is numerically equal to the current required to produce a deflection of 45° in it.
   $\theta = 45° \Rightarrow I = K \tan(45°) \Rightarrow I = K$
   SI unit of 'K' $\Rightarrow$ ampere
3. It doesn't work at magnetic poles as $B_H = 0$
4. Error can be **minimized** by **Increasing** Length of magnetic needle
5. Sensitivity and accuracy of tangent galvanometer is maximum when the deflection is $\Rightarrow \theta = 45°$. Error is minimum when $\theta = 45°$

### Illustration 103:

A tangent galvanometer has a coil of 50 turns and a radius of 20 cm, the horizontal component of the earth's magnetic field is $B_H = 3 \times 10^{-5}$ T. Find the current which gives a deflection of 45°.

### Solution:

We have $i = K \tan\theta = \dfrac{2RB_H}{\mu_0 N} \tan\theta = \dfrac{2 \times (0.20\text{m}) \times (3 \times 10^{-5}\text{T})}{4\pi \times 10^{-7}\,\text{TmA}^{-1} \times 50} \tan 45° = 0.19\text{A}$

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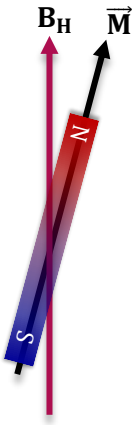
## Magnetometer

It is an instrument used to compare the horizontal component of magnetic field of earth of two different places, to compare magnetic fields and magnetic moments of two bar magnets. It is also called oscillation magnetometer.

**Working Principle : Angular SHM**

This device works on the principle, that whenever a freely suspended bar magnet in earth horizontal magnetic field component ($B_H$) is slightly disturbed from its equilibrium position then, it will experience a torque and execute angular S.H.M. (Rotation is possible only in horizontal plane.)




When a dipole is suspended in a uniform magnetic field it will align itself parallel to field. Now if it is given a small angular displacement θ with respect to its equilibrium position. The restoring torque acts on it :-

$\tau = -MB_H \sin\theta$

For small angle, $\sin\theta \approx \theta$

$\tau = -MB_H\theta$

For angular SHM, $\tau = -I\omega^2\theta$

$\Rightarrow\ -I\omega^2\theta = -MB_H\theta$

$$\Rightarrow \omega = \sqrt{\frac{MB_H}{I}} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{MB_H}{I}}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{I}{MB_H}}$$

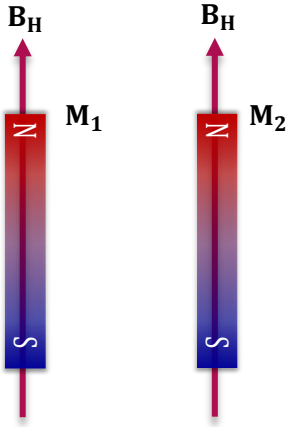
where :

M : magnetic moment of bar magnet

I : Moment of Inertia = $\frac{ml^2}{12}$ {m : mass of magnet, $\ell$ : length of magnet}

**Comparison of two magnets of same size (Same moment of inertia)**

Let the two magnets of same size have moment of inertia I and magnetic moments $M_1$ and $M_2$. Suspend these two given magnets one by one in the vibration magnetometer and note the time period in each case.




$$T_1 = 2\pi\sqrt{\frac{I}{M_1B_H}} \qquad ...(i)$$

$$T_2 = 2\pi\sqrt{\frac{I}{M_2B_H}} \qquad ...(ii)$$

Dividing equation (i) and (ii)

$$\frac{T_1}{T_2} = \sqrt{\frac{M_2}{M_1}} \text{ or } \frac{M_1}{M_2} = \frac{T_2^2}{T_1^2}$$

By knowing $T_1$ and $T_2$ the ratio $\frac{M_1}{M_2}$ can be determined.

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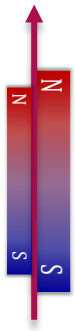
**Comparison of two magnets of different size (Different moment of inertia)**

Let the two magnets have moments of inertia $I_1$ and $I_2$ and magnetic moments $M_1$ and $M_2$ respectively. Place these two given magnets one upon the other as shown in Figure (a). This combination is called sum combination. This combination is put in the magnetometer and set it into oscillations. The time period of combination $T_1$ is determined.

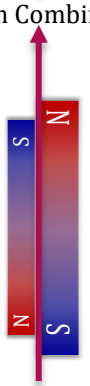


Sum Combination

$I_{net} = I_1 + I_2$ , $M_{net} = M_1 + M_2$

$$T_1 = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 + M_2)B_H}} \quad \text{...(i)}$$

Now, the two magnets are placed as shown in Figure (b). This combination is called 'difference combination'. This combination is put in the magnetometer and its time period $T_2$ is determined.



Difference Combination

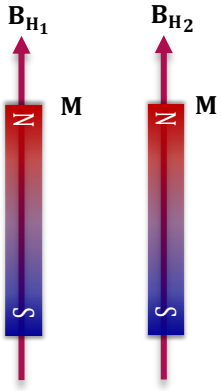
$I_{net} = I_1 + I_2$ , $M_{net} = M_1 - M_2$

$$T_2 = 2\pi\sqrt{\frac{I_1 + I_2}{(M_1 - M_2)B_H}} \quad \text{...(ii)}$$

Dividing, $\frac{T_1}{T_2} = \sqrt{\frac{M_1 - M_2}{M_1 + M_2}}$ [from equation (i) and (ii)]

**Comparison of earth's magnetic field at two different places (different $B_H$)**

Let the vibrating magnet have moment of inertia I and magnetic moment M. Let it be vibrated at places, where earth's magnetic field is $B_{H_1}$ and $B_{H_2}$




Then, $T_1 = 2\pi\sqrt{\frac{I}{MB_{H_1}}}$ and $T_2 = 2\pi\sqrt{\frac{I}{MB_{H_2}}}$

$T_1$ and $T_2$ are determined by placing magnetometer at two different places, one by one.

Dividing, $\frac{T_1}{T_2} = \sqrt{\frac{B_{H_2}}{B_{H_1}}}$

$$\Rightarrow \frac{T_1^2}{T_2^2} = \frac{B_{H_2}}{B_{H_1}} \Rightarrow \frac{T_1^2}{T_2^2} = \frac{B_2\cos\theta_2}{B_1\cos\theta_1}$$

$$\frac{B_1}{B_2} = \frac{T_2^2\cos\theta_2}{T_1^2\cos\theta_1}$$

By knowing $T_1$, $T_2$ and $\theta_1$, $\theta_2$ the ratio $\frac{B_1}{B_2}$ can be determined.

**Important point:**

(1) If we cut any bar magnet into **n equal parts normal to its length**, then time period of vibration in external magnetic field-

$$T' = \frac{T}{n}$$

(2) If we cut any bar magnet into **n equal parts along its length**, then time period of vibration in external magnetic field-

$$T' = T$$

**Illustration 104:**

At a certain place a magnet makes 30 oscillations per minute. At another place where the magnetic field is double, its time period will be

**Solution:**

$$T = 2\pi\sqrt{\frac{I}{MB_H}}; \therefore \frac{T_1}{T_2} = \sqrt{\frac{(B_H)_2}{(B_H)_1}} \Rightarrow T_2 = T_1\sqrt{\frac{(B_H)_1}{(B_H)_2}}$$

Here $n_1$ = 30 oscillation /min $= \frac{1}{2}$ oscillation/sec

$$\therefore\ T_1 = \frac{1}{n_1} = 2\,\text{sec}$$

$$\therefore\ T_2 = 2\sqrt{\frac{B_H}{2B_H}} = 2\times\frac{1}{\sqrt{2}} = \sqrt{2}\,\text{sec}$$

## BEGINNER'S BOX-10

**1.** A cell of an emf of 2V and internal resistance of 0.5 Ω is sending current through a tangent galvanometer of resistance 4.5Ω. If another external resistance of 95 Ω is introduced, the deflection of galvanometer is 45°. Reduction factor of galvanometer is :-

(1) 0.02 A (2) 0.01 A (3) 0.04 A (4) 0.05 A

**2.** Two tangent galvanometers A and B connected in series to a d. c. source have their number of turns in the ratio 1 : 3 and diameters in the ratio 1 : 2 of their coils which is correct for reduction factor and deflection of both galvanometers.

(1) $K_A < K_B, \theta_A > \theta_B$ (2) $K_A = K_B, \theta_A = \theta_B$ (3) $K_A > K_B, \theta_A < \theta_B$ (4) $K_A > K_B, \theta_A > \theta_B$

**3.** The time period of a freely suspended bar magnet does not depends on :-

(1) The length of the magnet

(2) The pole strength of the magnet

(3) The horizontal component of the earth's magnetic field

(4) The length of the suspension fibre

**4.** For a vibration magnetometer, the time period of freely suspended bar magnet can be reduced by:-

(1) Moving it towards south pole (2) Moving it towards north pole

(3) Moving it towards equator (4) None

**5.** A bar magnet is freely suspended in such a way that, when it oscillates in the horizontal plane. It makes 20 oscillations per minute at a place, where dip angle is 30° and 15 oscillations per minute at a place, where dip angle is 60°. Ratio of total earth's magnetic field at these two places :-

(1) $9\sqrt{3}:16$ (2) $9:\sqrt{3}$ (3) $\sqrt{3}:16$ (4) $16:9\sqrt{3}$

**6.** Magnetic moments of two identical bar magnets are M and 2M respectively. Both are combined in such a way that their similar poles are together. The time period in this situation is '$T_1$'. If polrity of one of the magnet is reversed then its period becomes '$T_2$' then :-

(1) $T_1 > T_2$ (2) $T_1 < T_2$ (3) $T_1 = T_2$ (4) None of these

**7.** Time period of thin bar magnet of vibration magnetometer is 'T'. If it cuts into two equal parts in then the time period of each part fashions.

(a) Along or parallel to its length (b) Perpendicular to its length,

(1) T, T (2) T,T/2 (3) T/2, T (4) T/2, T/2

## Neutral Points

### Neutral Points around a Bar Magnet

It is a point where net magnetic field is zero.

At this point magnetic field of bar magnet or current carrying coil or current carrying wire is just neutralised by horizontal component of earth magnetic field ($B_H$).

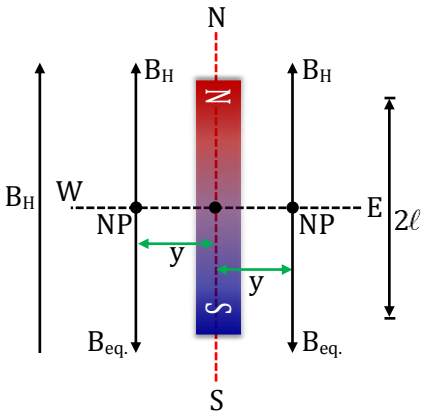
A compass needle placed at neutral point then it may set itself in any direction.

Location of Neutral Points :

**(a) When N–pole of magnet directed towards North :-**

Two neutral points symmetrically located on equatorial line of magnet. Let distance of each neutral point from centre of magnet is 'y' then

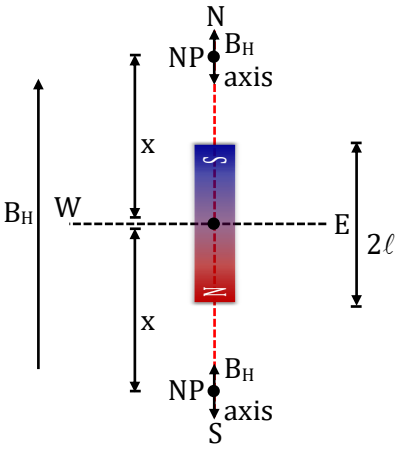
$$B_{eq} = B_H \Rightarrow \frac{\mu_0}{4\pi} \cdot \frac{M}{y^3} = B_H \quad (\text{If } y >>> \ell)$$




**(b) When S–pole of magnet directed towards North :-**

Two neutral points symmetrically located on the axial line of magnet. Let distance of each neutral point from centre of the magnet is x, then

$$B_{axis} = B_H \Rightarrow \frac{\mu_0}{4\pi} \frac{2M}{x^3} = B_H \ (\text{If } x >>> \ell)$$




### Illustration 105:

The earth's magnetic field at the equator is approximately 0.4 G. Estimate the earth's dipole moment.

**Solution:**

Equatorial magnetic field $B_E = \frac{\mu_0 M}{4\pi r^3}$

Magnetic moment $M = \frac{B_E(4\pi r^3)}{\mu_0}$

where $B_E = 0.4G = 4 \times 10^{-5}T$, $r = 6.4 \times 10^6$ m

$$= \frac{4\times10^{-5}\times(6.4\times10^6)^3}{\mu_0/4\pi} = 4 \times 10^2 \times (6.4 \times 10^6)^3$$

$$= 1.04 \times 10^{23} \text{ A–m}^2$$

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## BEGINNER'S BOX-11

**1.** The ratio of the magnetic field due to a small bar magnet in end on position to broad side on position is :-

(1) $\frac{1}{4}$ (2) $\frac{1}{2}$ (3) 1 (4) 2

**2.** The magnetic field at a point X on the axis of a small bar magnet is equal to the field at a point Y on the equator of the same magnet. The ratio of the distances of point X and Y from the centre of the magnet is :-

(1) $2^{-3}$ (2) $2^{-1/3}$ (3) $2^{3}$ (4) $2^{1/3}$

**3.** If a wire in the earth's magnetic field carries a current vertically downwards, it is possible to obtain a neutral point :-

(1) North of the wire (2) South of the wire (3) East of the wire (4) West of the wire

**4.** A short bar magnet placed on the table along the north south line. Its north pole is directed towards geographic north then location of neutral points is :-

(A) North (B) South (C) East (D) West

(1) A and B (2) C and D (3) A and D (4) B and C

**5.** A short magnet of moment 6.75 A–$m^2$ produces neutral points on its axis. If the horizontal component of earth's magnetic field $5 \times 10^{-5}$ Wb/$m^2$, then the distance of the neutral points from the centre of magnet:-

(1) 10 cm (2) 20 cm (3) 30 cm (4) 40 cm

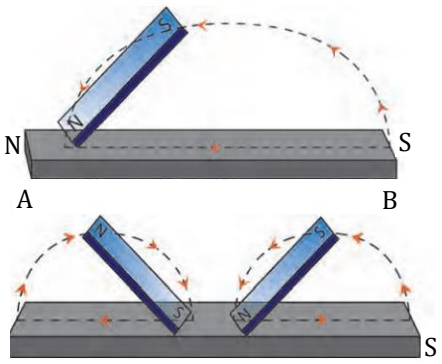
**6.** A straight wire carries current in vertical upward direction. A point 'P' lies just east of it at a small distance and another point 'Q' lies to the west just same distance. The magnetic field at these points :– (consider earth magnetic field)

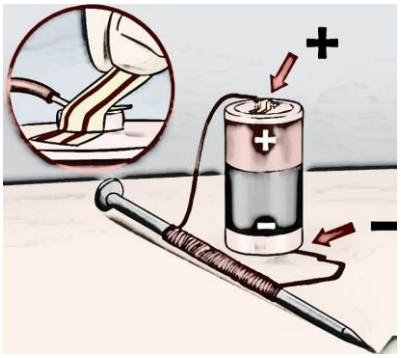
(1) $B_P > B_Q$ (2) $B_P < B_Q$ (3) $B_P = B_Q$ (4) None

## Magnetization

Magnetization, also termed as magnetic polarization, is a vector quantity that gives the measure of the density of permanent or induced dipole moment in a given magnetic material.

The magnetic effects of a material can also be induced by passing an electrical current through the material; the magnetic effect is caused by the motion of electrons in atoms, or the spin of the electrons or the nuclei.

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**Magnetizing field or Magnetic Intensity $(\vec{H})$**

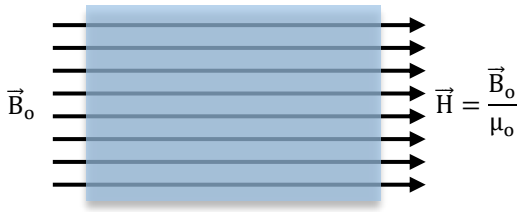
Field in which a material is placed for magnetization, called as magnetizing field.

$\vec{H} = \frac{\vec{B}_o}{\mu_o}$

unit of $\vec{H}$ : A/m

$\vec{B}_o$ : Magnetic Field

$\mu_o$ : Permeability of free space




**Intensity of magnetization $(\vec{I})$**

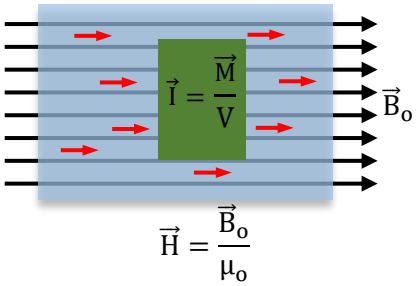
When a magnetic material is placed in magnetising field then induced dipole moment per unit volume of that material is known as intensity of magnetisation.

$\vec{I} = \frac{\vec{M}}{V}$

unit of $\vec{I}$ : A/m $\left\{\because \frac{\vec{M}}{V} = \frac{I\vec{A}}{V} = \frac{\text{ampere} \times \text{meter}^2}{\text{meter}^3}\right\}$

Induced magnetic field due to these induced dipoles in the material is given by:

$\vec{B}_{in} = \mu_o \vec{I}$




**Magnetic susceptibility ( $\chi_m$ )**

$\chi_m = \frac{I}{H}$ [It is a scalar with no units & dimensions]

Physically it represents the ease with which a magnetic material can be magnetised.

**Note:-** A material with more $\chi_m$ , can be change into magnet easily.

**Magnetic Permeability ( $\mu$ )**

$\mu = \frac{B}{H} = \frac{\text{Total magnetic field inside a material}}{\text{Magnetizing field}}$

Unit of $\mu$ : $\mu = \frac{B_m}{H} \equiv \frac{Wb/m^2}{A/m} = \frac{\text{Weber}}{A-m} = \frac{H-A}{A-m} = \frac{H}{m}$

It represents the degree up to which a material can be penetrated by the magnetic field lines

**Relative Permeability ($\mu_r$)**

$\mu_r = \frac{\mu}{\mu_0}$

It has no units and dimensions.

**Relation between $\mu$ and $\chi_m$**

When a magnetic material is placed in a magnetic field $\vec{B}_o$ for magnetization, then total magnetic field in the material is

$\vec{B}_m = \vec{B}_o + \vec{B}_{in}$ {$\vec{B}_{in}$= induced field.}

$\because \vec{B}_0 = \mu_0 \vec{H}$ ; $\vec{B}_{in} = \mu_0 \vec{I}$

$\therefore \vec{B}_m = \mu_o(\vec{H} + \vec{I})$

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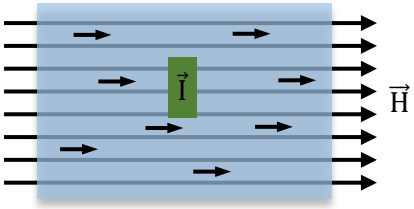
$B_m = \mu_o(H + I)$

$\mu H = \mu_o H\left(1 + \frac{I}{H}\right)$

$\mu = \mu_o(1 + \chi_m)$

$\frac{\mu}{\mu_o} = (1 + \chi_m)$

$\mu_r = (1 + \chi_m)$




For vacuum, $\mu_r = 1$ , $\chi_m = 0$

For air, $\mu_r = 1.04$ , $\chi_m = 0.04$

**Illustration 106:**

A solenoid of 200 turns/m is carrying a current of 5A. Relative permeability of core material of solenoid is 3000. Determine the magnitudes of the magnetic intensity, magnetization and the magnetic field inside the core.

**Solution:**

magnetic intensity $H = ni = 200\ m^{-1} \times 5A = 1000\ Am^{-1}$

$\mu_r = 1 + \chi$, i.e., $\chi = 3000 - 1 = 2999$

Hence, the magnetization $I = \chi H = 2.999 \times 10^6\ Am^{-1}$

The magnetic field $B = \mu_0\mu_r H$

$B = 3000\ \mu_0 H = 3000 \times 4\pi \times 10^{-7} \times 1000 = 3.769\ T$

**Illustration 107:**

When a rod of magnetic material of size 10 cm × 0.5 cm × 0.2 cm is located in magnetising field of $0.5 \times 10^4$ A/m then a magnetic moment of 5 A-$m^2$ is induced in it. Find out magnetic induction in rod.

**Solution:**

Total magnetic induction $B = \mu_0 (I + H) = \mu_0\left(\frac{M}{V} + H\right)$ $\left(\because I = \frac{M}{V}\right)$

$$= 4\pi \times 10^{-7}\left(\frac{5}{10^{-6}} + 0.5 \times 10^4\right) = 6.28\ Wb/m^2$$

**Illustration 108:**

A rod of magnetic material of cross section 0.25 $cm^2$ is located in 4000 A/m magnetising field. Magnetic flux passes through the rod is $25 \times 10^{-6}$ Wb. Find out for rod

(i) permeability (ii) magnetic susceptibility (iii) magnetisation

**Solution:**

(i) Magnetic flux $\phi = BA \Rightarrow B = \phi/A = \frac{25 \times 10^{-6}}{0.25 \times 10^{-4}} = 1\ Wb/m^2$

$B = \mu H \Rightarrow \mu = \frac{B}{H} = \frac{1}{4000} = 2.25 \times 10^{-4}\ Wb/A\text{-}m$

(ii) $\mu_r = 1 + \chi_m$ $\chi_m = \mu_r - 1$ $\left(\because \mu_r = \frac{\mu}{\mu_0}\right)$

$\chi_m = \frac{\mu}{\mu_0} - 1 = \left(\frac{2.5 \times 10^{-4}}{4\pi \times 10^{-7}} - 1\right) = 199 - 1 = 198$

(iii) Magnetization $I = \chi_m H = 198 \times 4000 = 7.92 \times 10^5\ A/m$

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# Magnetic Materials

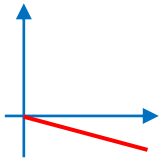
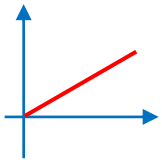
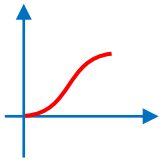
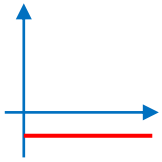
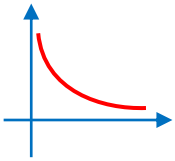
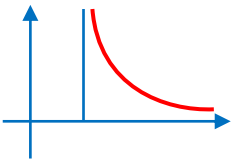
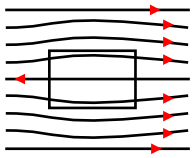
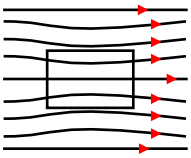
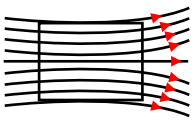
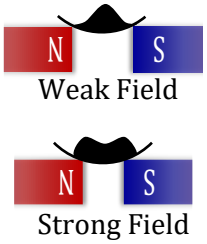
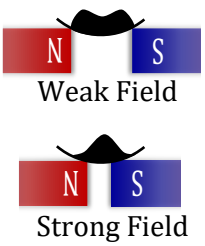
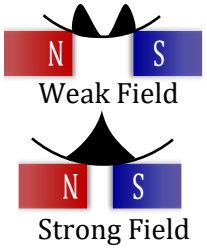
## Types of Magnetic Materials

On the basis of magnetic properties of the materials [as magnetization intensity (I), Susceptibility ($\chi_m$) and relative permeability ($\mu_r$)] Faraday divide these materials in three classes –

Based on the observation of how materials respond to external magnetizing field

| | Diamagnetic | Paramagnetic | Ferromagnetic |
|---|---|---|---|
| Cause of Magnetism | Orbital Motion of electrons | Spin Motion of electrons | formation of Domains |

### Classification of Magnetic Materials

| Properties | Diamagnetic | Paramagnetic | Ferromagnetic |
|---|---|---|---|
| I | Small, negative | Small, positive | Very large, positive |
| $\chi_m$ | Small, negative | Small, positive | Very Large, positive |
| $\mu_r$ | $1 > \mu_r > 0$ | $2 > \mu_r > 1$ | $\mu_r \gg 1$ |
| I – H curve | I → Small, negative varies linearly with field  | I → Small, positive, varies linearly with field  | I → very large, positive varies non-linearly with field  |
| Variation of $\chi_m$ with Temp | $\chi_m$ → small, negative & temperature independent $\chi_m \propto T^0$  | $\chi_m$ → small, positive & varies inversely with temp. Curie law $\chi_m \propto \frac{1}{T}$  | $\chi_m$ → very large, positive & temp. dependent Curie Weiss law (for $T > T_C$) $\chi_m \propto \frac{1}{T-T_c}$ ($T_C$ = Curie temperature)  |
| Degree of magnetization |  |  |  |
| Behavior in non-uniform magnetizing field |   |   |   |

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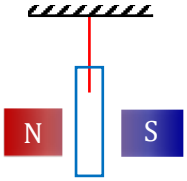
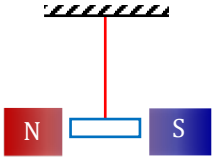
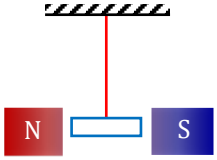
| | | | |
|---|---|---|---|
| Magnetic moment of single atom | Atoms do not have any permanent magnetic moment | Atoms have permanent magnetic moment which are randomly oriented. (i.e. in absence of external magnetic field the magnetic moment of whole material is zero) | Atoms have permanent magnetic moment which are organised in domains. |
| When rod of material is suspended between poles of magnet. | It becomes perpendicular to the direction of external magnetic field.   | If there is strong magnetic field in between the poles then rod becomes parallel to the magnetic field.   | Weak magnetic field between magnetic poles can made rod parallel to field direction.   |
| Magnetic moment of substance in presence of external magnetic field | Value $\vec{M}$ is very less and opposite to $\vec{H}$. | Value $\vec{M}$ is low but in direction of $\vec{H}$. | $\vec{M}$ is very high and in direction of $\vec{H}$. |
| Examples | Bi, Cu, Ag, Pb, $H_2O$, Hg, $H_2$, He, Ne, Au, Zn, Sb, NaCl, Diamond. (May be found in solid, liquid or gas). | Na, K, Mg, Mn, Sn, Pt, Al, $O_2$ (May be found in solid, liquid or gas.) | Fe, Co, Ni all their alloys, $Fe_3O_4$ Gd, Alnico, etc. (Normally found only in solids) (crystalline solids) |

**Note:** Above Curie temperature, ferromagnetic materials lose their permanent magnetic properties and behave as paramagnetic materials.

**Illustration 109:**

If a ferromagnetic material is inserted in a current carrying solenoid, the magnetic field of solenoid

**Solution:**

The flux inside the solenoid increases due to the presence of ferromagnetic material and hence the field of solenoid increases largely.

### Illustration 110:

Susceptibility of Mg at 300 K is $1.2 \times 10^{-5}$. The temperature at which susceptibility will be $1.8 \times 10^{-5}$ is

### Solution:

$$\chi \propto \frac{1}{T}$$

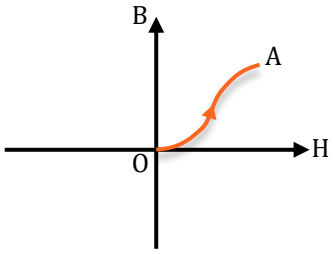
$\therefore \quad \chi_1 T_1 = \chi_2 T_2$

Hence, $T_2 = \frac{1.2\times10^{-5}\times300}{1.8\times10^{-5}} = 200\,K$

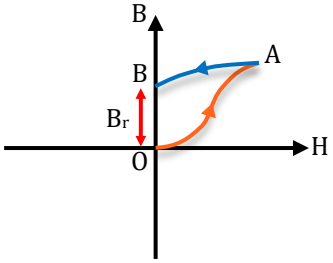
### Magnetic Hysteresis

Only Ferromagnetic materials show magnetic hysteresis, when Ferromagnetic material is placed in external magnetic field for magnetization.

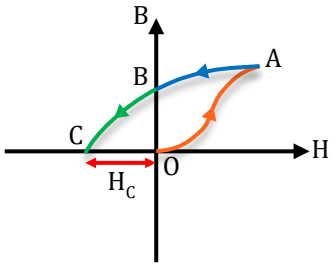
- When magnetizing field is increased across a ferromagnetic material, the magnetic field inside the material also increases.
  If magnetising field is increased further there will be no change in the magnetic field inside the material.




- When this magnetising field is reduced, magnetic field inside the material decreases (A to B).
  When this magnetizing field is removed, there is some residual magnetic field in the material known as **Retentivity ($B_r$)**




- To remove this residual magnetic field inside the material, external magnetizing field is again applied but in opposite direction till net magnetic field inside the material becomes zero (B to C).
  This magnetizing field required to destroy residual magnetic field is known as **Coercivity ($H_C$)**.



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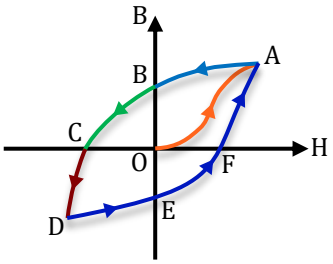
- As external magnetizing field is further increased, magnetic field again develops in the material (C to D).
  Now if magnetising field is again decreased then graph repeats itself again i.e., magnetic field decreases, and it achieves the residual value (D to E). Now if magnetizing field is reversed and increased then magnetic field becomes zero (E to F). If the magnetizing field is further increased then again magnetic field starts rising (F to A).
- Due to lagging behind of B with H this curve is known as hysteresis curve.
  [Lagging of B behind H is called hysteresis]




**Cause of hysteresis :**

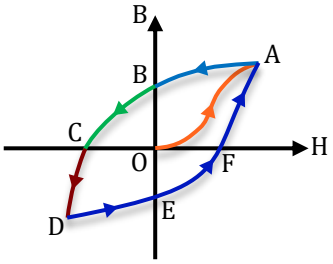
By removing external magnetizing field (H = 0), the magnetic moment of some domains remains aligned in the applied direction of previous magnetizing field which results into a residual magnetism.

**Residual magnetism (OB) = $B_r$** (retentivity/remanence) :

Retentivity of a specimen is a measure of the magnetic field remaining in the ferromagnetic specimen when the magnetizing field is removed.

**Coercivity (OC) = $H_c$ :**

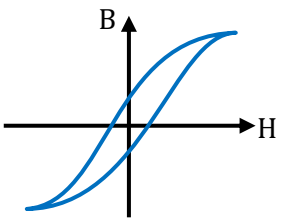
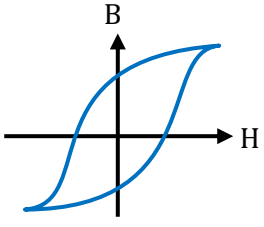
Coercivity is a measure of magnetizing field required to destroy the residual magnetism of the ferromagnetic specimen.




**Hysteresis Loss**

- The area of hysteresis loop is equal to the energy loss per cycle per unit volume.
- Its value is different for different materials.
- The work done per cycle per unit volume of material is equal to the area of hysteresis loop.
  Total energy loss in material = $W_H$
  $W_H$ = volume of material × area of hysteresis curve × frequency × time.
  $W_H = V \times A \times n \times t$ joule
  $W_H = \frac{V \times A \times n \times t}{J}$ calorie

## Soft and Hard Magnetic Materials

| Soft magnetic materials | Hard magnetic materials |
|---|---|
| 1. Low retentivity<br>2. Low coercivity<br>3. Small hysteresis Loss<br>4. Suitable for making electromagnets, cores of transformers etc.<br>e.g. Soft Iron<br>  | 1. High retentivity<br>2. High coercivity<br>3. Large hysteresis Loss<br>4. suitable for permanent magnet<br>e.g. Steel, Alnico<br>  |

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### Illustration 111:

Which of the following statement is True of False about hysteresis

(i) This effect is common to all ferromagnetic substances.

(ii) The hysteresis loop area is proportional to the thermal energy developed per unit volume of the material

(iii) The hysteresis loop area is independent of the thermal energy developed per unit volume of the material.

(iv) The shape of the hysteresis loop is characteristic of the material.

### Solution:

(i) True (ii) True (iii) False (iv) True

The energy lost per unit volume of a substance in a complete cycle of magnetisation is equal to the area of the hysteresis loop.

### Illustration 112:

The area of hysteresis loop of a material is equivalent to 250 Joule/$m^3$. When 10 kg material is magnetised by an alternating field of 50Hz then energy lost in one hour will be if the density of material is 7.5 gm/$cm^3$.

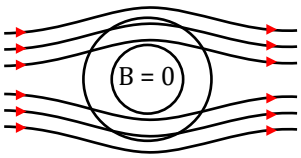
### Solution:

Energy loss in 1 hour = Area × Volume × time × frequency = $\frac{250\times10\times3600\times50}{7500}$ = 60000 J = $6 \times 10^4$ J

## Magnetic Shielding and Electromagnet

**Magnetic Shielding**

When a soft iron ring is placed in magnetic field, most of the lines are found to pass through the ring and no lines pass through the space inside the ring. The inside of the ring is thus protected against any external magnetic effect. This phenomenon is called magnetic screening or shielding.



Iron Ring in external magnetic field

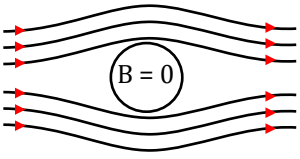
It is used to protect costly wrist–watches and other instruments from external magnetic fields by enclosing them in a soft–iron case or box.

**Special Note :.**

**(i)** Superconductors also provide perfect magnetic screening due to exclusion of lines of force. This effect is called **'Meissner effect'**

**(ii)** Relative magnetic permeability of super conductor is zero as the susceptibility is $\chi = -1$.

So, we can say that super conductors behave like **perfect diamagnetic material**.



Super conductor in a field

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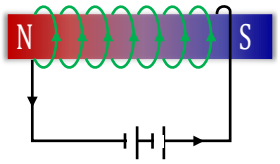
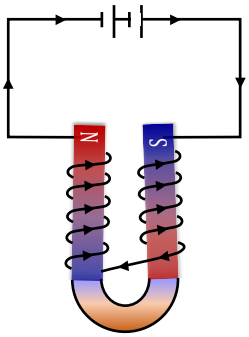
**Electromagnet**

- An electromagnet can be defined as a magnet which functions on electricity.
- A current carrying solenoid behaves like a bar magnet.




(a)




(b)

- On passing current through this solenoid, a magnetic field is produced in the space within the solenoid.
- If we place a soft iron rod in the solenoid, the magnetism of the solenoid increases hundreds of times and the solenoid is called an electromagnet.
- It is a temporary magnet.
- An electromagnet is made by winding closely a number of turns of insulated copper wire over a soft iron straight rod or a U-shaped rod.

**Applications of Electromagnet**

(i) Electromagnets are used in electric bell, electric lock, telephone diaphragms etc.

(ii) In medical field they are used in extracting bullets from the human body.

(iii) Large electromagnets are used in cranes for lifting and transferring big machines and large objects.

**Illustration 113:**

For protecting a magnetic needle, it should be placed in an iron box or in wooden box

**Solution:**

In an iron box

## BEGINNER'S BOX-12

**1.** Fill in the blanks

Diamagnetism is the ..... property of every material and it is generated due to mutual interaction between the applied magnetic field and ..... motion of electrons.

**2.** Match the column I with column II -

| Column-I | | Column-II | |
|---|---|---|---|
| (A) | Paramagnetic substance | (p) | $\chi_m = -1$ |
| (B) | Diamagnetic substance | (q) | $\chi_m < 0$ |
| (C) | Super conductor | (r) | $\chi_m >> 1$ |
| (D) | Ferromagnetic substance | (s) | $\mu_r > 1$ |

**3.** How the magnetic susceptibility ($\chi_m$) of diamagnetic, paramagnetic and ferromagnetic substances depends on temperature (T).

**4.** A solenoid has $10^3$ turns per unit length. On passing a current of 2A, magnetic induction is measured to be $4\pi$ Wb/m$^2$. Calculate magnetic susceptibility of core.

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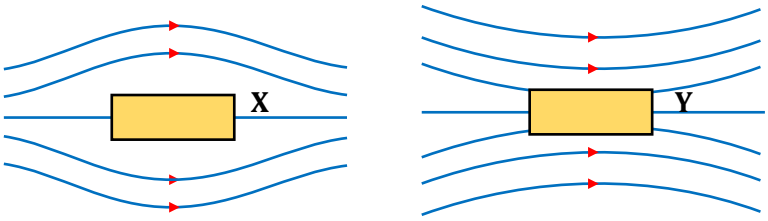
**5.** A magnetising field of 1600 A-m$^{-1}$ produces a magnetic flux of 3.14× 10$^{-5}$ weber in a bar of iron of cross-section 0.2 cm$^2$ . Calculate permeability and susceptibility of the bar.

**6.** A uniform magnetic field gets modified as shown below, when the specimans X and Y are placed in it.

(a) Identify the two specimans X and Y

(b) State the reason for the behaviour of the field lines in X and Y.




**7.** How will a dia., para. and a ferromagnetic material behave when kept in a non–uniform external magnetic field? Give two examples of each of these materials. Name two main characteristics of a ferromagnetic material which help us to decide its suitability for making

(a) a permanent magnet (b) an electromagnet.

Which of these two characteristics should have high or low values for each of these two types of magnets?

**8.** An iron rod of cross sectional area 4 cm$^2$ is placed with its length parallel to a magnetising field 1600 Am$^{-1}$. The flux through the rod is 4 × 10$^{-4}$ weber. Find out permeability of the material of the rod?

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## BEGINNER'S BOX — ANSWERS KEY

### BEGINNER'S BOX-1

**1.** (2) **2.** (3) **3.** (4) **4.** (2) **5.** (1)
**6.** (2) **7.** (4) **8.** (1) **9.** (1) **10.** (2)
**11.** (3) **12.** (3) **13.** **(a)** 2 **(b)** 1 **14.** **(a)** 3 **(b)** 1 **(c)** 2

### BEGINNER'S BOX-2

**1.** (1) **2.** (4) **3.** (3) **4.** (1) **5.** (1)

**6.** **i.** (a) $\frac{\mu_0 I}{4R}\otimes$ (b) $\frac{\mu_0 I}{8R}\otimes$ **ii.** (a) $\frac{3\mu_0 I}{8R}\odot$ (b) $\frac{\mu_0 I}{8R}\odot$

**iii.** (a) $\frac{\mu_0 I}{2R}\left(1+\frac{1}{\pi}\right)\odot$ (b) $\frac{\mu_0 I}{2R}\left(1-\frac{1}{\pi}\right)\otimes$ **iv.** (a) $\frac{\mu_0 I}{2R}\left(\frac{1}{2}+\frac{1}{\pi}\right)\otimes$ (b) $\frac{\mu_0 I}{2R}\left(\frac{1}{2}-\frac{1}{\pi}\right)\odot$

**v.** (a) $\frac{\mu_0 I}{8R}\left(3+\frac{2}{\pi}\right)\odot$ (b) $\frac{\mu_0 I}{4R}\left(1+\frac{2}{\pi}\right)\odot$ **vi.** (a) $\frac{3\mu_0 I}{8R}\otimes$ (b) $\frac{\mu_0 I}{24R}\odot$

**7.** (4) **8.** (2) **9.** $\frac{R}{\sqrt{150}}$

### BEGINNER'S BOX-3

**1.** (1) **2.** (2) **3.** (3) **4.** (4) **5.** (3)

### BEGINNER'S BOX-4

**1.** (a) True (b) True **2.** B **3.** 1T

### BEGINNER'S BOX-5

**1.** (4) **2.** (2) **3.** (1) **4.** (3)
**5.** (a) 1 : 2 : 2 (b) 2 : 2 : 1 (c) $1:\sqrt{2}:1$ (d) $1:\sqrt{2}:\sqrt{2}$
**6.** (a) 2 : 1 : 1 (b) 1 : 1 : 2 (c) 2 : 1 : 2 (d) 2 : 1 : 1
**7.** (a) $C_p < C_e$ (b) $C_p = C_e$ (c) $C_p < C_e$ (d) $C_p < C_e$ (C = curvature)
**8.** (a) 1 : 2 : 2 (b) 2 : 1 : 1 (c) 2 : 1 : 1 **9.** 1.2 cm, 4.35 cm
**10.** 0.1m, $6.28\times 10^{-7}$ sec **11.** T **12.** (a), (b) and (c)
**13.** (a), (b) and (c) **14.** (a), (b), (c) and (d)
**15.** (a), (b) and (c)
**16.** (3) **17.** (2) **18.** (1) **19.** (3) **20.** (2)
**21.** (2) **22.** (2) **23.** (4)

### BEGINNER'S BOX-6

**1.** (2) **2.** (1) **3.** (1) **4.** (4) **5.** (1)
**6.** (2) **7.** (1) **8.** (4) **9.** (1)

### BEGINNER'S BOX-7

**1.** (a) 2M, (b) Zero, (c) $\sqrt{2}$ M, (d) M,(e) Zero, (f) $2\sqrt{2}$M **2.** (a) $\frac{M}{2}$, (b) $\frac{M}{2}$

**3.** (a) $\frac{M}{\sqrt{2}}$, (b) $\frac{M}{2}$, (c) $\frac{2M}{\sqrt{2}}$, (d) Zero **4.** $\frac{\pi a^3 B}{2\sqrt{2}\mu_0}$

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**5.** (a) $\sqrt{3}\frac{I\ell^2}{36}$, (b) $\frac{I\ell^2}{16}$, (c) $\frac{\sqrt{3}}{24}I\ell^2$, (d) $\frac{I\ell^2}{4\pi}$

**6.** 2 : 1 **7.** One turn **8.** $\sqrt{2}M$

## BEGINNER'S BOX-8

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | (3) | **2.** | (2) | **3.** | (4) | **4.** | (4) | **5.** | (1) |
| **6.** | (1) | **7.** | (2) | **8.** | (2) | **9.** | (3) | **10.** | (1) |

## BEGINNER'S BOX-9

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **1.** | (2) | **2.** | (3) | **3.** | (3) | **4.** | (1) |

## BEGINNER'S BOX-10

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | (1) | **2.** | (3) | **3.** | (4) | **4.** | (3) | **5.** | (4) |
| **6.** | (2) | **7.** | (2) | | | | | | |

## BEGINNER'S BOX-11

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** | (4) | **2.** | (4) | **3.** | (3) | **4.** | (2) | **5.** | (3) |
| **6.** | (1) | | | | | | | | |

## BEGINNER'S BOX-12

**1.** Intrinsic, orbital **2.** A – s, B – q, C – p, D – r

**3.** For diamagnetic $\chi_m \propto T^0$

For paramagnetic $\chi_m \propto \frac{1}{T}$

For ferromagnetic $\chi_m \propto \frac{1}{T - T_c}$ here $T_c \rightarrow$ Curie temperature

**4.** 4999 **5.** Permeability ($\mu$) = $9.8\times 10^{-4}\frac{Wb}{A-m}$ Susceptibility ($\chi_m$) = 779.25

**6.** (a) X $\rightarrow$ Dia-magnetic Y $\rightarrow$ Para-magnetic

(b) Magnetic permeability of dia-magentic speciman is slightly less than vacuum and magnetic permeability of para–magnetic speciman is slightly more than vacuum.

**7.** **Diamagnetic** : Moves (very weakly) away from strong field region towards the weak field region.

**Para magnetic** : Moves (weakly) towards the strong field region.

**Ferromagnetic** : Moves (strongly) towards the strong field region.

**Two examples** : Diamagnetic (Bismuth, Copper)

Paramagnetic (Aluminium, Oxygen)

Ferromagnetic (Iron, Nickel)

Two characteristics of ferromagnetic materials : Coercivity and Retentivity

For Permanent Magnets : High Coercivity and High Retentivity

For Electromagnets : Low Coercivity and Low Retentivity

**8.** $\mu = 0.625 \times 10^{-3}$ Wb / A-m